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MARKDOWN PRICING STRATEGY UNDER A DUAL-CHANNEL SUPPLY CHAIN WITH STRATEGIC CONSUMERS

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Abstract. This study investigates the markdown pricing strategies for a manufacturer and a retailer in a two-period dual-channel supply chain, where the manufacturer sells its products *via* its own direct channel and an independent retail channel to strategic consumers who may wait for markdowns. A two-period game is developed to systematically study the optimal regular prices and markdown prices under four cases, *i.e.*, no markdown in both channels, markdown only in the direct channel, markdown only in the retail channel, and markdowns in both channels. By comparing the different cases, we find that the manufacturer benefits most from the case with markdowns in both channels, where the markdown rate of the retail channel is lower than that of the direct channel. On the other hand, the results indicate that the retailer may also profit most from the case with markdowns in both channels when the consumer acceptance of the direct channel is sufficiently high; otherwise, the retailer enjoys the highest profit under the case with markdown only in the retail channel. Finally, it is found that strategic consumer behavior has a positive impact on the retailer's profit but a negative impact on the manufacturer's profit.

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1. Introduction

Development of technologies and advances in e-commerce greatly change manufacturers' and retailers' sales modes and shape consumers' purchase patterns. Consumers today are accustomed to shopping *via* multiple channels according to their needs. This triggers manufacturers to utilize online or direct channels to offer consumers more convenient access to products in addition to traditional retail channels [7]. In the following, we use online channel and direct channel interchangeable. According to a survey from U.S. Department of Commerce, online sales in the United States increased by 14.2% to 870.8 billion in 2021 compared to that in 2020, and accounted for 13.2% of total sales. Moreover, the rapid innovations speed up product updates and shorten product life cycles, some products become obsolete quickly after new products are introduced. Despite the use of the manufacturer's direct channel and an independent retail channel to increase sales, a surplus of

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products in these two channels is generally encountered, which leads to higher storage cost and management cost. In order to address the overstock problem, manufacturers and retailers usually adopt a markdown pricing to attract consumers' attention on the outdated products [1,26,43]. For instance, some top electronic companies, like Apple, Huawei, and Samsung, usually mark down prices of old-generation phones in their retail and online channels when they launch new versions of products. A steep discount is also applied by fashion retailers, such as Nike, Gap and Zara, to liquidate the excess inventory in their retail and online channels [5]. Therefore, markdown pricing plays an essential role in liquidating inventory of old-version products.

With great popularity of markdown pricing among companies or retailers, some consumers have been trained to wait for sales to take advantage of a future discount rather than buy early at a regular price. A survey by BestBuy shows that an estimated 20% of consumers strategically wait for late purchases in order to get a substantial discount [23]. Recently, an experimental study even reports that 77% of consumers would wait for a future discount rather than buying immediately [27]. However, the waiting behavior is not always favorable for the consumers as the products maybe be sold out before a discount is applied due to limited inventory. Therefore, the consumers need to rationally weigh options between purchasing in the current period and in the future *via* predicting price markdowns. In the literature, this type of consumer is usually referred to as strategic consumers, and the corresponding purchase behavior is called strategic consumer behavior [8, 34]. The impact of strategic consumer behavior on firms has been widely investigated in past decades (see [35] for a detailed review). However, most studies indicate that the strategic consumer behavior is bad for the firms' profitability because the consumers will choose to purchase product at a low price (*e.g.*, [9, 19, 45]). Consequently, the firms should trade-off between the benefit of clearing inventory and the loss due to strategic consumer behavior when considering whether to implement a markdown pricing.

On the other hand, some scholars have examined the optimal markdown pricing in the presence of strategic consumers. For example, Baucells *et al.* [3] built an axiomatic model to solve a markdown management problem when consumers make a wait-or-buy decision. Hao *et al.* [14] developed a framework to study the joint stocking and markdown policy for an innovating firm facing strategic consumers. Mersereau and Zhang [24] explored markdown pricing when the fraction of strategic consumers is uncertain. Extensive investigations focus on markdown pricing under either a single-echelon supply chain or a traditional supply chain setting where a manufacturer sells products to the end consumers through a retailer. Zhang *et al.* [47] explored the effect of strategic consumers on the business model choice between the reselling and the agency models under a markdown pricing policy. Different from their investigations, this paper considers markdown pricing under a two-period dual-channel supply chain with strategic consumers, where the manufacturer sells its products through a direct channel and an independent retailer to consumers with strategic behavior in two periods. Furthermore, this work studies whether the manufacturer and the retailer adopt the markdown pricing *via* the direct channel and the retail channel, investigates the optimal markdown decisions at each scenario, and find the best markdown pricing strategy for the manufacturer and the retailer.

Motivated by the above discussion, we address the following three questions: (1) What is the optimal regular prices and markdown prices for the direct channel and the retail channel under different markdown pricing scenarios? (2) Is it beneficial for the manufacturer and the retailer to implement a markdown pricing and what is the optimal markdown pricing strategy for the manufacturer and the retailer? (3) How do consumers' strategic behavior and the consumer acceptance of the direct channel influence the optimal pricing decisions and the profits of the manufacturer and the retailer? To the best of our knowledge, this paper is the first to investigate the markdown decisions and related competitive pricing strategy under a two-period dual-channel supply chain setting. The results summarized from this paper provide the optimal regular prices and the markdown prices for the manufacturer and the retailer under different markdown pricing strategies and shed light on the choice of the optimal markdown pricing strategy, which offers decision support on operating a dual-channel supply chain in the presence of strategic consumers.

The study has been undertaken by developing a two-period dual-channel supply chain model, where products are sold *via* both the manufacturer's direct channel and an independent retail channel in two periods to strategic consumers. Before the selling period, the manufacturer and the retailer announce the markdown pricing strategy to the strategic consumers, which can be with or without markdown. As the products are sold over two periods in two channels, there are four possible scenarios: (1) no markdown in both channels; (2) markdown in the direct channel only; (3) markdown in the retail channel only; (4) markdowns in both channels. In the first period, the manufacturer sells regular-priced products *via* a dual-channel supply chain, where the manufacturer determines wholesale price for the retailer and direct price for end consumers, and then the retailer sets the retail price for end consumers in the retail channel. By observing the prices, the consumers determine whether to purchase the product immediately or wait for sales in the next period if the markdown pricing strategy is adopted by the manufacturer or/and the retailer. If choosing the former, the consumers also need to select either direct channel or retail channel to purchase the products.

In the second period, if the manufacturer adopts the markdown pricing strategy, the manufacturer announces the markdown price in the direct channel. On the other hand, if the retailer adopts the markdown pricing strategy, the manufacturer decides the wholesale price and then the retailer determines the markdown price in the retail channel. After knowing markdown price of the product, the consumers who do not purchase the product in the first period decide from which channel to buy the product; otherwise, they leave without purchases. To account for the effect of strategic consumer behavior on purchasing from each channel at different periods, we follow the works of Cachon and Swinney [4], Liu and Zhang [20] and Shum *et al.* [32] by using a time discount factor to represent consumer's patience level in their individual utility. Consumers have rational expectations for the markdown prices in the direct and retail channel. As a result, they can anticipate the prices charged by the manufacturer and the retailer in both periods when assessing the payoffs of each potential purchase.

Based on the analysis on the different markdown pricing strategies, we observe several interesting results. First, we derive the optimal regular prices and markdown prices in a two-period dual-channel supply chain, where the manufacturer or/and the retailer implement a markdown pricing through the direct channel or/and the retail channel, respectively. The results provide the manufacturer and the retailer with pricing decision support in the face of different markdown scenarios. Second, by comparing the aforementioned four markdown pricing scenarios, we find that the manufacturer is the most beneficial from the scenario with the markdowns in both channels, where the markdown rate of the retail channel is lower than that of the direct channel. From the perspective of the retailer, the results indicate that when the consumer acceptance of the direct channel is sufficiently high, the retailer may also benefit the most from the scenario with markdowns in both channels; otherwise the retailer enjoys the highest profit in the markdown only in the retail channel. Combined with the profits of the retailer and the manufacturer, the results show that the markdowns in both channels is the most beneficial for the whole dual-channel supply chain because the manufacturer plays a dominant role under the supply chain. Thirdly, compared with the case with markdowns in both channels, the retailer offers consumers a lower discount under the case with markdowns only in retail channel unless consumers are sufficiently strategic or the consumer acceptance level of direct channel is low, while the manufacturer provides a higher discount for consumers under the case with the markdown only in the direct channel for certain conditions. Finally, it is found that strategic consumer behavior has a positive impact on the retailer's profit but a negative impact on the manufacturer's profit.

This paper is organized as follows. Related literature is reviewed in Section 2 and the model description is presented in Section 3. The models for four different markdown pricing scenarios are analyzed in Section 4, and the numerical results and comparative results are presented in Section 5. Finally, conclusions and future research are presented in Section 6.

2. Literature review

Combined with our research problem, we will briefly review three streams of literature related to our study in the following: (1) strategic consumer behavior; (2) markdown pricing; (3) two-period dual-channel supply chain.

Strategic consumer behavior has been extensively concerned by researchers in a study of operations management. Coase [8] was the first to study the strategic consumers behavior in durable goods market, where consumers delay their purchases due to anticipation lower prices in the future. Recently, some researchers have set out to widely explore strategic consumer behavior from different perspectives, such as product pricing [38, 42], competition [20], inventory management [29, 37], and quick response [4]. Most literature focus on the effect of the strategic consumers on a single retailer without investigating the interaction between players in a vertical supply chain. Su and Zhang [34] studied the impact of strategic consumer behavior on supply chain performance, and find that a centralized supply chain may perform strictly worse than a decentralized supply chain when consumers are strategic. Farshbaf-Geranmayeh and Zaccour [10] developed a multi-period model to investigate pricing and advertising decisions in a supply chain, where the retailer chooses to advertise the product in a market. The result shows that an integrated supply chain would adopt the same pricing policy as in the decentralized case. Chen *et al.* [6] explored the impact of the reference price on the supply chain in presence of strategic consumers, and showed that the centralized structure may not always be optimal relative to a decentralized structure. Different from the above literature, we consider the impact of strategic consumer behavior on pricing decisions in two periods for a dual-channel supply chain with different markdown pricing strategies.

Another stream related to our research is the markdown pricing. Yin *et al.* [43] proposed a preannounced markdown pricing strategy to investigate alternative inventory display formats for a retailer who sells a limited inventory of product over a finite selling season. Whang [36] examined a markdown pricing for a retailer who announces a pair of declining prices for two selling periods. Hu *et al.* [15] used a stochastic dynamic multi-period inventory model to address joint ordering and markdown decisions problem. Smith and Agrawal [33] proposed a markdown problem in a retail chain with multiple nonidentical stores, and analyzed the impact of inventory dependence of demand on the pricing, inventory allocation, and store consolidation. Shin *et al.* [31] used a newsvendor model to explore the optimal markdown timing and order quantity under a preannounced discount rate, and found the markdown timing that is beneficial for the retailer is harmful to the manufacture under certain conditions. Furthermore, some scholars consider the markdown in a competitive market, Sen [30] studied the effects of competition on markdown timing decisions of two firms with fixed inventories. Adida and zer [1] investigated whether and why retailers can use markdown as an effective defense against a competitor entering the market with everyday low price when consumers show availability regret and high-price regret. While multiple studies on markdown pricing have been conducted in the literature, they focus mainly on a markdown pricing strategy in traditional supply channel. In this paper we enrich the current literature by taking into account all types of markdown pricing in a dual-channel supply chain structure.

Our work is also related to the literature on a two-period dual-channel supply chain. Dual-channel supply chain with a single-period has been widely studied in the past few decades (*e.g.*, [2,13,21,22,48]). In a recent, a multi-period dual-channel have been paid attention to some researchers. Xiong *et al.* [39] provided a two-period dual-channel model, where a manufacturer sells a durable product directly through both a direct channel and an independent dealer who adopts a mix of selling and leasing. Yan *et al.* [40] further explored the effect of product durability on manufacturer encroachment in a two-period dual-channel supply chain. Zhang *et al.* [46] explored two-stage pricing strategies of a dual-channel supply chain considering public green preference. However, there is a lack of current literature examining the impact of strategic consumer behavior on the performance of dual-channel supply chain. This study is conducted to fill this gap.

Our paper has some contributions to the literature. The topic is relatively similar to the research of Yin *et al.* [44] and Lei *et al.* [18]. Yin *et al.* [44] examined a two-period dual-channel supply chain model where the manufacturer adopts two wholesale pricing strategies to sell its product *via* one retailer or two distinct retailers in two periods in the presence of strategic consumers. They assumed that the direct price matches the retail price in each period, but the price competition in dual-channel supply chain was ignored. In contrast, we discuss product prices in both channels can be different in any selling period and the manufacturer sells the product *via* both own direct channel and an independent retailer in each period, which is more realistic. On the other hand, Lei *et al.* [18] examined the impact of strategic consumer behavior on preannounced pricing under a two-period dual-channel supply chain, where the discount price is only available in the direct channel. Different their studies, we focus on a dynamic pricing strategy in a dual-channel supply chain with markdown strategy in the direct channel and/or the retail channel.

3. Problem definition, notations and assumptions

This section mainly presents the problem definition, related notations and assumptions used to develop the mathematical models, and the timing of game is also described in this section.

3.1. Problem definition

We consider a dual-channel supply chain where a manufacturer provides a product for end consumers *via* its own direct channel and an independent retail channel over two periods, where the direct channel and retail channel are indexed by m and r, respectively. Let c_m and c_r denote the operation costs of the direct channel and the retail channel, respectively. For the direct channel, the manufacturer sells the product to the end consumers through his direct channel at a direct price p_{im}, where $i = \{1, 2\}$ denotes the selling period. Meanwhile, for the retail channel, the manufacturer first sells the product to the retailer at a wholesale price ω_i, and then the retailer sells the product at a retail price p_{ir} to end consumers. Besides the pricing decisions, the manufacturer and the retailer need to determine individually whether to adopt the markdown pricing through the direct channel and the retail channel in the second period, respectively. Therefore, there exist four possible scenarios to sell the products. The first scenario is the basic model without markdown in both channels (Case 1), which is widely investigated as summarized in the literature. The second scenario is the markdown in the direct channel only (Case 2). Many manufacturers offer a deep discount price in their own direct channels to attract consumers for near-obsolete inventory, while the retail channel does not carry out the same markdown activities. The third scenario is the markdown in the retail channel only (Case 3). A typical example is "Black Friday" in the United States and "Boxing Day" in Canada, when retailers offer various markdowns to entice shopping physical traffic. The last scenario is markdowns in both channels (Case 4). As some important traditional holidays, such as the Christmas and New Year, provide people more leisure time and desire for shopping, these impel the manufacturer to markdown prices in the retail and direct channels to attract more consumers in order to promote sales.

Consumers have a heterogeneous valuation v that follows a uniform distribution on $[0, 1]$ in two periods and two channels [6, 7]. In the first period, when consumers purchase a product from the retail channel, they can evaluate the product on-site and buy it immediately. Hence, the consumer's utility from her purchases in the retail channel is $u_{1r} = v - p_{1r}$. However, the consumer's utility for purchase from direct channel is $u_{1m} = \beta v - p_{1m}$, where β is called as the consumer acceptance of the direct channel, which is incurred by some inconvenience factors, such as the lack of examining the products in person, long delivery time and so on [7]. Kacen *et al.* [17] investigated a survey and find that most products are less acceptable from a direct channel than a retail channel. Therefore, $0 < \beta < 1$ is assumed in this paper. Consumers are rational and strategic in deciding their purchase time if the markdown is available in the second period. Consumer's utility in the second period will be discounted by α ($0 \le \alpha \le 1$). In our model, α not only captures the opportunity cost of delaying purchasing [28] , but also meaures the degree of consumer's patience [4, 20, 32]. A higher α represents that the consumers are more patient and willing to postpone their purchases. In the special case of $\alpha = 0$, the consumers are myopic, namely, the consumer purchases the product as long as the consumer's utility is positive. Thus, the consumer's utility purchasing from retail channel in the second period is $u_{2r} = \alpha(v - p_{2r})$ and from direct channel in the second period is $u_{2m} = \alpha(\beta v - p_{2m})$.

3.2. Notations and assumptions

The model in this study is formulated based on the following notations and assumptions. The notations used in this paper are summarized in Table 1, and the main assumptions proposed to formulate analytical models are listed as follows:

Table 1. Summary of notations.

Indexes	Description
k	Superscript, index different cases, where $k = \{1, 2, 3, 4\}$ refers to no markdown in both channels, markdown in the direct channel only, markdown in the retail channel only, markdowns in both channels, respectively.
$*$	Superscript, index the equilibrium solution.
i	Subscript, index different selling periods, where $i = \{1, 2\}$
Decision variables	
ω_i^k	Wholesale price in period i under Case k
p_{im}^k	Direct channel price in period i under Case k
p_{ir}^k	Retail price in period i under Case k
Parameters	
α	The consumer's patience level, where $0 \leq \alpha \leq 1$
β	The consumer acceptance of the direct channel, where $0 < \beta < 1$
c_r	The operation cost of the retail channel
c_m	The operation cost of the direct channel
θ_k	The marginal valuation under Case k
D_{im}^k	The demand of the direct channel in period i under Case k
D_{ir}^k	The demand of the retail channel in period i under Case k
Π_m^k	The total profit of the manufacturer under Case k
Π_r^k	The total profit of the retailer under Case k
Π^k	The total profit of the supply chain under Case k
M^k	The markdown rate under Case k

1. All consumers arrive at the beginning of period 1, and each consumer buys at most one product.
2. We assume that $0 < c_m < c_r < 1$, which is attributed to the fact that the retailer would incur a higher cost for products than the direct channel due to expensive store rents, high holding and handling costs of inventory, and lots of staff salaries, which is consistent with Chiang *et al.* [7] and Yang *et al.* [41]. Without any loss of generality, the production cost of the manufacturer is normalized to zero.
3. Channel prices must exceed marginal costs such that $p_{ir} \geq \omega_i \geq c_r \geq 0$ and $p_{im} \geq c_m \geq 0$; otherwise, there is no profit in the retail and direct channels. Meanwhile, we assume that $p_{im} \geq \omega_i$, which assures that the retailer buys the product from the manufacturer rather than the direct channel.
4. Following the work of Liu and Zhang [20], we make an assumption $\beta > \alpha$ in this paper, implying that the attractiveness of buying a product immediately is quite strong since consumers prefer enjoying the product sooner rather than waiting to buy later, although the markdown price induces consumers to delay their purchases for the product at a lower price. In particular, the preference of consumer for purchasing the product from the direct channel in the first period is higher than waiting for a markdown from the retail channel in the second period when the product have the same net present price in two periods.

3.3. Timing of game

The sequence of events corresponding to the four different markdown pricing scenarios is summarized in Figure 1. Before the selling period, the manufacturer and the retailer determine whether to adopt the markdown pricing in the direct channel and the retail channel, respectively. All consumers arrive at the market. In the first period, the manufacturer as a leader decides on the wholesale price ω_1 to the retailer and simultaneously announces the direct price p_{1m} for the end consumers. Then, the retailer as a follower determines the retail price p_{1r} to end consumers. Given the first-period selling price, consumers decide to buy in the first period or wait to buy in the next period based on their knowledge of the selling prices in two periods. If choosing to

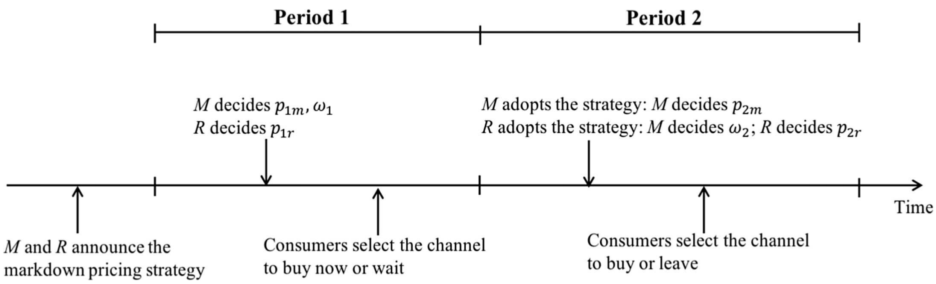

Figure 1. Decision-making process.

buy immediately, the consumers further determine from which channel to purchase the product. In the second period, the manufacturer determines the wholesale price ω_2 to the retailer if the retailer adopts markdown pricing through the retail channel, and the direct price p_{2m} to the end consumers if the markdown pricing is available in the direct channel. The retailer then sets retail price p_{2r} to maximize her profit under the existing of markdown in the retail channel. The consumers who decide to wait for purchases either choose purchase channel based on the markdown strategy or leave the market.

4. Model framework and equilibrium solution

In this section, we investigate the equilibrium pricing of the two-period Stackelberg game through backward induction approach under four different markdown scenarios: (1) no markdown in both channel; (2) markdown in the direct channel only; (3) markdown in the retail channel only; (4) markdowns in both channels. Backward induction in game theory is an iterative process of reasoning backward in time, from the end of a problem to solve finite extensive form and sequential games, and infer a sequence of optimal actions [25]. In the process of problem solving, backward induction first determines the optimal strategy of the player who makes the last move in the game. Then, the optimal action of the next-to-last moving player is determined, taking the last player's action as given. This process continues backward until the best action for every point in time has been determined. Based on the method, all proofs are presented in Appendix.

4.1. No markdown in both channels (Case 1)

We consider the strategy without markdown in both channels, *i.e.*, the retailer and the manufacturer sell the product at the same price in two periods. Since consumers are rational and strategic, they prefer to buy the products sooner rather than later when expecting no a markdown in the second period. Hence, we only need to discuss the first-period optimization. The consumer's utility purchasing from the retail channel is $u_{1r} = v - p_{1r}$ and from the direct channel is $u_{1m} = \beta v - p_{1m}$. Consumers will buy the product from the retail channel if $u_{1r} \geq 0$ and from the direct channel if $u_{1m} \geq 0$. Hence, the indifference valuation of purchasing or not purchasing from the channel can be derived as $v_r = p_{1r}$ in retail channel and $v_m = \frac{p_{1m}}{\beta}$ in direct channel. Also, the indifferent valuation of purchasing the product between the two channels is $\theta_1 = \frac{p_{1r} - p_{1m}}{1-\beta}$ by setting $u_{1r} = u_{1m}$. If $v_m \leq v_r$, then $v_m \leq v_r \leq \theta_1$. Hence, if $\theta_1 \leq 1$, both channels have positive demands, *i.e.*, $D^1_{1r} = 1 - \theta_1$ in retail channel and $D^1_{1m} = \theta_1 - v_m$ in direct channel; if $\theta_1 \geq 1$, there is no demand in the retail channel, and the direct channel's demand is $D^1_{1m} = 1 - v_m$. If $v_m \geq v_r$, then $\theta_1 \leq v_r \leq v_m \leq 1$. There is no demand in the direct channel, and the retail channel's demand is $D^1_{1r} = 1 - v_r$. We summarize the demands of two channels as follows:

$$(D^1_{1r}, D^1_{1m}) = \begin{cases} ((1 - p_{1r}), 0) & \omega_1 \leq p_{1r} < \frac{p_{1m}}{\beta} \\ (1 - \frac{p_{1r} - p_{1m}}{1-\beta}, \frac{p_{1r} - p_{1m}}{1-\beta} - \frac{p_{1m}}{\beta}) & \frac{p_{1m}}{\beta} \leq p_{1r} < p_{1m} + 1 - \beta \\ (0, 1 - \frac{p_{1m}}{\beta}) & p_{1m} + 1 - \beta \leq p_{1r} \leq 1 \end{cases} \tag{4.1}$$

In this paper, we focus on the situation where the demands for two channels are non-negative. Hence, $\frac{p_{1m}}{\beta} \leq p_{1r} < p_{1m} + 1 - \beta$ is assumed to be satisfied. The corresponding profits of two channels under Case 1 can be given as follows:

$$\begin{aligned}&\Pi_r^1 = (p_{1r} - \omega_1 - c_r)(1 - \tfrac{p_{1r}-p_{1m}}{1-\beta}),\\&\Pi_m^1 = \omega_1(1 - \tfrac{p_{1r}-p_{1m}}{1-\beta}) + (p_{1m} - c_m)(\tfrac{p_{1r}-p_{1m}}{1-\beta} - \tfrac{p_{1m}}{\beta}).\end{aligned} \tag{4.2}$$

By using the backward induction approach, we can find out that the optimal pricing decisions for the manufacturer and the retailer, as shown in Theorem 4.1.

Theorem 4.1. *Under the scenario without markdown in both channels, the optimal prices are given by*

$$p_{1r}^{*1} = \tfrac{3-\beta+c_m+c_r}{4}, p_{1m}^{*1} = \tfrac{\beta+c_m}{2} \text{and} \omega_1^{*1} = \tfrac{1-c_r}{2}. \tag{4.3}$$

Theorem 4.1 elaborates that the selling prices of the retailer and manufacturer are determined by the consumer acceptance of the direct channel and operation costs in two channels, as there is no markdown in period 2. The results show that when consumers increasingly prefer the direct channel, the manufacturer will increase the direct price to earn more profit, while the retailer will decrease her retail price to attract more consumers.

4.2. Markdown in the direct channel only (Case 2)

In two periods, the manufacturer reduces the direct price in the second period, while the retailer keeps the retail price unchanged in the second period, namely, $p_{2m} \leq p_{1m}$ and $p_{2r} = p_{1r}$. Since consumers prefer to buy the products sooner rather than later and expect that there is no the markdown pricing in the retail channel, there are no consumer purchases from the retail channel in the second period. Thus, in the second period, we only consider the direct channel's optimization problem. Let θ_2 be the marginal valuation at which a consumer is indifferent between purchasing in period 1 and period 2, namely, a consumer with $v \leq \theta_2$ chooses to purchase in period 2, and with $v \geq \theta_2$ purchases the product in period 1. Therefore, the consumer who purchases in the second period has the product valuation over $[0, \theta_2]$, where $\theta_2 \in [0, 1]$. Given the marginal valuation θ_2, a consumer with valuation $v \leq \theta_2$ purchases product from the direct channel if and only if $\alpha(\beta v - p_{2m}) \geq 0$. Thus, we can obtain the demand for the direct channel as $D_{2m}^2 = \theta_2 - \frac{p_{2m}}{\beta}$ and the profit of the manufacturer in period 2 is $\Pi_{2m}^2 = (p_{2m} - c_m)(\theta_2 - \frac{p_{2m}}{\beta})$. Obviously, the optimal direct price in the second period is $p_{2m}^{*2} = \frac{\beta\theta_2 + c_m}{2}$. Correspondingly, the profit of the manufacturer is $\Pi_{2m}^{*2} = \frac{(\beta\theta_2 - c_m)^2}{4}$.

Next, we analyze the optimization problem in period 1. Consumer with valuation $v \geq \theta_2$ faces whether to buy the product immediately or wait in the second period, and which channel to purchase from if she chooses the former. Given the marginal valuation θ_2 and the optimal second-period direct price (p_{2m}^{*2}), the demand functions are obtained *via* a discussion of consumer purchase decision. A consumer with valuation $v \geq \theta_2$ purchases product from the retail channel if $v - p_{1r} \geq \beta v - p_{1m}$,$v - p_{1r} \geq \alpha(\beta v - p_{2m}^{*2})$ and $v - p_{1r} \geq 0$ hold. Similarly, A consumer purchases product from the direct channel if $\beta v - p_{1m} \geq v - p_{1r}$, $\beta v - p_{1m} \geq \alpha(\beta v - p_{2m}^{*2})$ and $\beta v - p_{1m} \geq 0$ hold. To ensure the two channels coexistence in period 1, the marginal valuation θ_2 must satisfy (1) $\beta\theta_2 - p_{1m} = \alpha(\beta\theta_2 - p_{2m}^{*2})$, *i.e.*, $\theta_2 = \frac{2p_{1m}}{\beta(2-\alpha)}$; (2) $\theta_2 \leq \frac{p_{1r}-p_{1m}}{1-\beta}$, *i.e.*, $p_{1r} \geq \frac{p_{1m}(2-\alpha\beta)}{\beta(2-\alpha)}$; (3) $\frac{p_{1r}-p_{1m}}{1-\beta} \leq 1$, *i.e.*, $p_{1r} \leq 1 - \beta + p_{1m}$. The first two conditions indicates the direct channel in period 1 have positive sales. The last condition indicates that the demand in retail channel is positive. Therefore, the demands of two channels in period 1 are

$$D_{1r}^2 = 1 - \tfrac{p_{1r}-p_{1m}}{1-\beta} \text{ and } D_{1m}^2 = \tfrac{p_{1r}-p_{1m}}{1-\beta} - \theta_2. \tag{4.4}$$

Meanwhile, the profits of the retailer and the manufacturer are expressed as follows, respectively:

$$\begin{aligned}&\Pi_r^2 = (p_{1r} - \omega_1 - c_r)(1 - \tfrac{p_{1r}-p_{1m}}{1-\beta}),\\&\Pi_m^2 = \omega_1(1 - \tfrac{p_{1r}-p_{1m}}{1-\beta}) + (p_{1m} - c_m)(\tfrac{p_{1r}-p_{1m}}{1-\beta} - \theta_2) + \Pi_{2m}^{*2}(\theta_2),\\&\text{where } \theta_2 = \tfrac{2p_{1m}}{\beta(2-\alpha)}.\end{aligned} \tag{4.5}$$

Theorem 4.2. *When only the direct channel adopts a markdown, the optimal prices are given by*

$$\begin{aligned} p_{1r}^{*2} &= \frac{(\beta-c_m)(2\alpha^2-2\alpha-1)-(3+c_r)(2\alpha-3)}{4(3-2\alpha)},\\ p_{1m}^{*2} &= \frac{\alpha^2(\beta-c_m)+2(2\beta+c_m-2\alpha\beta)}{2(3-2\alpha)},\\ \omega_1^{*2} &= \frac{(1-\alpha)^2(\beta-c_m)+(1-c_r)(3-2\alpha)}{2(3-2\alpha)},\\ p_{2m}^{*2} &= \frac{\alpha(\beta+3c_m)-2(\beta+2c_m)}{2(2\alpha-3)}. \end{aligned} \tag{4.6}$$

From Theorem 4.2, we can directly observe that as the consumer acceptance of the direct channel increases, it is intuitive that the manufacturer will set high selling prices of the direct channel in two periods and the retailer sets a low retail price in the first period. Surprisingly, the manufacturer increases the procurement cost for the retailer, which aims to attract more consumers to purchase from the direct channel by decreasing the order in the retail channel.

4.3. Markdown in the retail channel only (Case 3)

Similar to Case 2, the markdown price is offered in the retail channel and the direct channel keeps the same direct price in two periods, namely, $p_{2r} \leq p_{1r}$ and $p_{2m} = p_{1m}$. Meanwhile, there are no sales for the direct channel in the second period. A consumer with $v \leq \theta_3$ will purchase the product from the retail channel if and only if $\alpha(v-p_{2r}) \geq 0$; otherwise the consumer will leave. Hence, the demand for the retail channel in the second period is $D_{2r}^3 = \theta_3 - p_{2r}$, where $\theta_3 \in [0,1]$ is the marginal valuation at which a consumer is indifferent between purchasing the product in the first period and second period. Correspondingly, the profits of two players are as follows:

$$\begin{aligned} \Pi_{2r}^3 &= (p_{2r}-\omega_2-c_m)(\theta_3-p_{2r}),\\ \Pi_{2m}^3 &= \omega_2(\theta_3-p_{2r}). \end{aligned} \tag{4.7}$$

Examining (4.7), the retailer's profit is maximized when $p_{2r}^{*3} = \frac{(\omega_2+\theta_3+c_r)}{2}$. Then the manufacturer's profit becomes $\frac{\omega_2(\theta_3-\omega_2-c_r)}{2}$. Therefore, the optimal wholesale price in the second period is $\omega_2^{*3} = \frac{\theta_3-c_r}{2}$. Substituting ω_2^{*3} into p_{2r}^{*3} and (4.7), we can obtian $p_{2r}^{*3} = \frac{3\theta_3+c_r}{4}$ and the optimal second-period profits of the retailer and the manufacturer as $\Pi_{2r}^{*3} = \frac{(\theta_3-c_r)^2}{16}$ and $\Pi_{2m}^{*3} = \frac{(\theta_3-c_r)^2}{8}$.

In period 1, the consumer has three purchase choices: (1) from the retail channel in period 1 or 2; (2) from the direct channel in period 1. To guarantee two demands coexistence in period 1, the retail price (p_{1r}) and the direct price (p_{1m}) need to satisfy (1) $\beta\theta_3 - p_{1m} = \alpha(\theta_3 - p_{2r}^{*3})$ (*i.e.*, $\theta_3 = \frac{4p_{1m}}{4\beta-\alpha}$) and $\theta_3 \leq \frac{p_{1r}-p_{1m}}{1-\beta}$ (*i.e.*, $p_{1r} \leq \frac{p_{1m}(4-\alpha)}{4\beta-\alpha}$), which manifests the demand for the direct channel positive; (2) $\frac{p_{1r}-p_{1m}}{1-\beta} \leq 1$ (*i.e.*, $p_{1r} \leq 1-\beta+p_{1m}$), which manifests the demand for the retail channel positive. Hence, the demands for both channels in the first period are given by

$$\begin{aligned} D_{1r}^3 &= 1-\frac{p_{1r}-p_{1m}}{1-\beta},\\ D_{1m}^3 &= \frac{p_{1r}-p_{1m}}{1-\beta}-\theta_3. \end{aligned} \tag{4.8}$$

Correspondingly, the manufacturer's and retailer's profits in the first period are obtained as follow:

$$\begin{aligned} &\Pi_r^3 = (p_{1r}-\omega_1-c_r)\left(1-\frac{p_{1r}-p_{1m}}{1-\beta}\right)+\Pi_{2r}^{*3}(\theta_3),\\ &\Pi_m^3 = \omega_1\left(1-\frac{p_{1r}-p_{1m}}{1-\beta}\right)+(p_{1m}-c_m)\left(\frac{p_{1r}-p_{1m}}{1-\beta}-\theta_3\right)+\Pi_{2m}^{*3}(\theta_3),\\ &\text{where } \theta_3 = \frac{4p_{1m}}{4\beta-\alpha}. \end{aligned} \tag{4.9}$$

Thus, the optimal pricing decisions for the two players in the first period are presented in Theorem 4.3 by using backward induction method.

Theorem 4.3. *When only the retail channel adopts the markdown, the optimal outcomes are given by*

$$\begin{aligned}
p_{1r}^{*3} &= \tfrac{(\alpha-4\beta)(\alpha c_r-\alpha-2\beta+2c_m+c_r+6)+\alpha c_r-3\beta-c_m+c_r+3}{4(2\alpha-8\beta+1)},\\
p_{1m}^{*3} &= \tfrac{(\alpha-4\beta)(\alpha c_r-\alpha+4\beta+4c_m)+4\beta c_r}{4(2\alpha-8\beta+1)},\\
\omega_1^{*3} &= \tfrac{(c_r-1)(\alpha^2-4\alpha\beta-4\alpha+18\beta-2)+2(\beta c_r-c_m)}{4(2\alpha-8\beta+1)},\\
p_{2r}^{*3} &= \tfrac{(5\alpha-8\beta+4)c_r+3(\alpha-4\beta-4c_m)}{4(2\alpha-8\beta+1)},\\
\omega_2^{*3} &= \tfrac{c_r(\alpha-8\beta)-\alpha+4\beta+4c_m}{2(8\beta-2\alpha-1)}.
\end{aligned} \tag{4.10}$$

Theorem 4.3 gives the optimal prices in two periods when the markdown is only adopted in the retail channel. Due to the intractability of the formulas, the impacts of the consumer's patience level and the consumer acceptance of direct channel on the optimal results are discussed by conducting a numerical experiment in next section.

4.4. Markdowns in both channels (Case 4)

In this case, the manufacturer and the retailer implement markdowns in the direct channel and the retail channel, respectively, *i.e.*, $p_{2m} \le p_{1m}$ and $p_{2r} \le p_{1r}$. Hence, consumers can buy the product through either the retail or direct channel in period 1 or the retail or direct channel in period 2 or leave. In the intertemporal choice, only the consumer with valuation $v \ge \theta_4$ will choose to buy the product in period 1, and $v \le \theta_4$ will wait to buy the product in period 2, where θ_4 is defined as the indifferent between purchasing in period 1 and period 2. Thus, in the second period, given θ_4, a consumer purchases the product from the retail channel if and only if $\alpha(v-p_{2r}) \ge \alpha(\beta v - p_{2m})^+$. Otherwise, the consumer chooses to purchase the product from the direct channel if $\alpha(\beta v - p_{2m}) \ge \alpha(v-p_{2r})^+$ or chooses not to purchase at all if $\beta v - p_{2m} < 0$. Similarly, to ensure the two channels' demands in period 2 are positive simultaneously, it is required that $\frac{p_{2m}}{\beta} \le \frac{p_{2r}-p_{2m}}{1-\beta} \le \theta_4$, *i.e.*,$\frac{p_{2m}}{\beta} \le p_{2r} < p_{2m} + (1-\beta)\theta_4$. The demands for the retail and the direct channels in Case 4 are $D_{2r}^4 = \theta_4 - \frac{p_{2r}-p_{2m}}{1-\beta}$ and $D_{2m}^4 = \frac{p_{2r}-p_{2m}}{1-\beta} - \frac{p_{2m}}{\beta}$, respectively. Hence, the profits of the retailer and the manufacturer in period 2 are expressed as below.

$$\begin{aligned}
\Pi_{2r}^4 &= (p_{2r}-\omega_2-c_r)(\theta_4 - \tfrac{p_{2r}-p_{2m}}{1-\beta}),\\
\Pi_{2m}^4 &= \omega_2(\theta_4 - \tfrac{p_{2r}-p_{2m}}{1-\beta}) + (p_{2m}-c_m)(\tfrac{p_{2r}-p_{2m}}{1-\beta} - \tfrac{p_{2m}}{\beta}).
\end{aligned} \tag{4.11}$$

By solving the optimization problem (4.11), we can derive the optimal pricing decisions and the profits of the retailer and the manufacturer in period 2 under Case 4, as presented in Lemma 4.4.

Lemma 4.4. *Given the consumers valuations in period 2 over* $[0,\theta_4]$, *the optimal pricing decisions in period 2 are given by*

$$\begin{aligned}
p_{2r}^{*4} &= \tfrac{\theta_4(3-\beta)+c_r+c_m}{4}, p_{2m}^{*4} = \tfrac{\theta_4\beta+c_m}{2}, \omega_2^{*4} = \tfrac{\theta_4-c_r}{2},\\
\Pi_{2r}^{*4} &= \tfrac{(\theta 4(1-\beta)-c_r+c_m)^2}{16(1-\beta)},\\
\Pi_{2m}^{*4} &= \tfrac{\beta(\beta\theta_4-c_m)^2+(-2c_r\beta+2c_m)(\beta\theta_4-c_m)-\beta(\theta_4-c_r)^2}{8\beta(1-\beta)}.
\end{aligned} \tag{4.12}$$

From (4.12), we can observe that consumer's strategic behavior does not affect the second-period optimal outcomes because of no opportunities to purchase in the future.

Based on the optimal second-period outcomes, we then analyze the game in the first period. Given the marginal valuation θ_4, the optimal second-period direct price (p_{2m}^{*4}) and the optimal second-period retail price (p_{2r}^{*4}), a consumer with valuation $v \ge \theta_4$ will purchase product from the retail channel in period 1 if $v - p_{1r} \ge (\beta v - p_{1m})^+$, $v - p_{1r} \ge \alpha(v - p_{2r}^{*4})$, and $v - p_{1r} \ge \alpha(\beta v - p_{2m}^{*4})$ hold; and from the direct channel in period 1 if $\beta v - p_{1m} \ge (v-p_{1r})^+$,$\beta v - p_{1m} \ge \alpha(v-p_{2r}^{*4})$ and $\beta v - p_{1m} \ge \alpha(\beta v - p_{2m}^{*4})$ hold. To guarantee the demand for the two channels positive simultaneously, the marginal valuation θ_4 must satisfy the following conditions: (1) $\beta\theta_4 - p_{1m} = \alpha(\theta_4 - p_{2r}^{*4})$ (*i.e.*, $\theta_4 = \frac{4p_{1m}}{4\beta-\alpha\beta-\alpha}$) and $\theta_4 \le \frac{p_{1r}-p_{1m}}{1-\beta}$, *i.e.*, $p_{1r} \le \frac{p_{1m}(4-\alpha\beta-\alpha)}{4\beta-\alpha\beta-\alpha}$, which assures the demand in direct channel is positive; (2) $\frac{p_{1r}-p_{1m}}{1-\beta} \leq 1$, which assures the demand in retail channel is positive. Therefore, the demands of the two channels in period 1 are given by $D_{1r}^4 = 1 - \frac{p_{1r}-p_{1m}}{1-\beta}$ and $D_{1m}^4 = \frac{p_{1r}-p_{1m}}{1-\beta} - \theta_4$. Correspondingly, the profits of the retailer and the manufacturer in period 1 are express as

$$\begin{aligned} &\Pi_r^4 = (p_{1r} - \omega_1 - c_r)(1 - \tfrac{p_{1r}-p_{1m}}{1-\beta}) + \Pi_{2r}^{*4}(\theta_4), \\ &\Pi_m^4 = \omega_1(1 - \tfrac{p_{1r}-p_{1m}}{1-\beta}) + (p_{1m} - c_m)(\tfrac{p_{1r}-p_{1m}}{1-\beta} - \theta_4) + \Pi_{2m}^{*4}(\theta_4), \\ &\text{where } \theta_4 = \tfrac{4p_{1m}}{4\beta-\alpha\beta-\alpha}. \end{aligned} \tag{4.13}$$

Solving the optimality (4.13) by the backward induction method, we can obtain the first-period optimal outcomes of the two players, which are given in the following Theorem 4.5.

Theorem 4.5. *When the retail and direct channels adopt the markdowns, the optimal outcomes are*

$$\begin{aligned} p_{1r}^{*4} &= \tfrac{(\alpha\beta+\alpha-4\beta)^2+(\alpha\beta+\alpha-4\beta)[6(\beta-1)-(c_r+c_m)(1+\alpha)]-(\beta+1)(\alpha c_m+\alpha c_r-3\beta-c_m+c_r+3)}{4(7\beta-2\alpha(1+\beta)-1)}, \\ p_{1m}^{*4} &= \tfrac{(\alpha\beta+\alpha-4\beta)^2-(\alpha\beta+\alpha-4\beta)[(1-\alpha)c_r-c_m(3+\alpha)]-\alpha(1+\beta(c_m+c_r))}{4(7\beta-2\alpha(1+\beta)-1)}, \\ \omega_1^{*4} &= \tfrac{(\alpha\beta+\alpha-4\beta)^2+(\alpha\beta+\alpha-4\beta)[4(\beta-1)+5c_r+c_m-\alpha(c_r+c_m)]-(1+\beta)(\alpha c_m+\alpha c_r-2\beta-2c_m-2c_r+2)}{4(7\beta-2\alpha(1+\beta)-1)}, \\ p_{2r}^{*4} &= \tfrac{(\alpha\beta+\alpha-4\beta)(\beta-2(c_m+c_r)-3)+(\beta-3)(c_m+c_r)\alpha-4(\beta c_m+2c_m-c_r)}{2(2\alpha(1+\beta)-7\beta+1)}, \\ p_{2m}^{*4} &= \tfrac{(\beta+2c_m)(\alpha\beta+\alpha-4\beta)+\alpha\beta(c_r+c_m)-2c_m\beta+\beta c_r+c_m}{2(2\alpha(1+\beta)-7\beta+1)}, \\ \omega_2^{*4} &= \tfrac{(2c_r-1)(\alpha\beta+\alpha-4\beta)-\alpha(c_m+c_r)+\beta c_r+3c_m}{2(7\beta-2\alpha(1+\beta)-1)}. \end{aligned} \tag{4.14}$$

Under this strategy, we not only get the optimal regular prices in the direct channel and the retail channel but also present the markdown prices in the two channels, which goes beyond the work of Yin *et al.* [44] who assumed the direct price was the same as the retail price in two periods. Due to the complexity of the problem and formulas, it is analytically intractable to derive the comparisons among four cases. Therefore, we will conduct a numerical study to gain a better understanding of the effect of the consumer's patience level and the consumer acceptance of the direct channel on the markdown pricing and the performance of the dual-channel supply chain.

According to the analysis of four cases above, when the manufacturer and the retailer adopt their pricing decisions in Stackelberg equilibrium, their profits can be obtained and summarized in the following Table 2.

5. Numerical experiments and comparative results

In this section, we perform several numerical studies to explore how the consumer's patience level (α) and the consumer acceptance of the direct channel (β) affect the optimal pricing decisions in two periods and the retailer's and the manufacturer's profits under four different scenarios. Furthermore, we compare the performances of the retailer, the manufacturer and the supply chain under the four different scenarios, and thereby gain additional insights into optimal markdown strategies. Based on existing literature and industrial practice, we set the values of parameters. First, we let $c_m = 0.1$ and $c_r = 0.2$ to ensure consistency with our assumptions shown in Subsection 3.2. Second, Huang *et al.* [16] showed that the consumer's patience level α is unlikely to be too low and confined the value of α to be $\alpha \in [0.5, 1]$. Therefore, the value of α can be taken as 0.7 in this paper. Finally, Kacen *et al.* [17] provided evidence that the range of the consumer acceptance of the direct channel (β) is from 0 and 1. Thus, we let $\beta = 0.8$, which is in line with our assumptions and also used by Yang *et al.* [41]. Referring to the data and assumptions, the parameters are set as $\alpha = 0.7, \beta = 0.8, c_m = 0.1$, and $c_r = 0.2$. In the following, we will examine the impact of α and β on the optimal results through holding the values of all other parameters unchanged.

5.1. Effects on prices in two periods

In this subsection, we explore the impacts of the consumer's patience level (α) and the consumer acceptance of the direct channel (β) on the retail prices, the direct prices and wholesale prices in two periods under four different cases.

TABLE 2. Optimal results under four cases.

	Case 1	Case 2	Case 3	Case 4
p_{1r}	$\frac{3-\beta+c_m+c_r}{4}$	$\frac{(\beta-c_m)(2\alpha^2-2\alpha-1+c_r)-(3+c_r)(2\alpha-3)}{4(3-2\alpha)}$	$\frac{(\alpha-4\beta)(\alpha c_r-\alpha-2\beta+2c_m+6)+\alpha c_r-3\beta-c_m+c_r+3}{4(2\alpha-8\beta+1)}$	$\frac{(\alpha\beta+\alpha-4\beta)^2+(\alpha\beta+\alpha-4\beta)[6(\beta-1)-(c_r+c_m)(1+\alpha)]-(\beta+1)(\alpha c_m+\alpha c_r-3\beta-c_m+c_r+3)}{4(7\beta-2\alpha(1+\beta)-1)}$
p_{1m}	$\frac{\beta+c_m}{2}$	$\frac{\alpha^2(\beta-c_m)+2(2\beta+c_m-2\alpha\beta)}{2(3-2\alpha)}$	$\frac{(\alpha-4\beta)(\alpha c_r-\alpha+4\beta+4c_m)+4\beta c_r}{4(2\alpha-8\beta+1)}$	$\frac{(\alpha\beta+\alpha-4\beta)^2-(\alpha\beta+\alpha-4\beta)[(1-\alpha)c_r-c_m(3+\alpha)]-\alpha(1+\beta(c_m+c_r))}{4(7\beta-2\alpha(1+\beta)-1)}$
ω_1	$\frac{1-c_r}{2}$	$\frac{(1-\alpha)^2(\beta-c_m)+(1-c_r)(3-2\alpha)}{2(3-2\alpha)}$	$\frac{(c_r-1)(\alpha^2-4\alpha\beta-4\alpha+18\beta-2)+2(\beta c_r-c_m)}{4(2\alpha-8\beta+1)}$	$\frac{(\alpha\beta+\alpha-4\beta)^2+(\alpha\beta+\alpha-4\beta)[4(\beta-1)+5c_r+c_m-\alpha(c_r+c_m)]-(1+\beta)(\alpha c_m+\alpha c_r-2\beta-2c_m-2c_r+2)}{4(7\beta-2\alpha(1+\beta)-1)}$
p_{2r}	–	–	$\frac{(5\alpha-8\beta+4)c_r+3(\alpha-4\beta-4c_m)}{4(2\alpha-8\beta+1)}$	$\frac{(\alpha\beta+\alpha-4\beta)(\beta-2(c_m+c_r)-3)\alpha+(\beta-3)(c_m+c_r)-4(\beta c_m+2c_m-c_r)}{2(-7\beta+2\alpha(1+\beta)+1)}$
p_{2m}	–	$\frac{\alpha(\beta+3c_m)-2(\beta+2c_m)}{2(2\alpha-3)}$	–	$\frac{(\beta+2c_m)(\alpha\beta+\alpha-4\beta)+\alpha\beta(c_r+c_m)-2c_m\beta+\beta c_r+c_m}{2(-7\beta+2\alpha(1+\beta)+1)}$
ω_2	–	–	$\frac{c_r(\alpha-8\beta)-\alpha+4\beta+4c_m}{2(8\beta-2\alpha-1)}$	$\frac{(2c_r-1)(\alpha\beta+\alpha-4\beta)-\alpha(c_m+c_r)+\beta c_r+3c_m}{2(7\beta-2\alpha(1+\beta)-1)}$
Π_r	$\frac{(\beta-1-c_m+c_r)^2}{16(1-\beta)}$	$\frac{(\beta-1-c_m+c_r)^2}{16(1-\beta)}$	$\frac{(\alpha-4\beta)^2[4(\beta-c_m)^2-(1-c_r)(8c_r+9\beta-8c_m-4)]2(\alpha-4\beta)[c_r(\beta-1)(2\alpha c_r-\alpha-4c_m)+2(c_r-1)(c_r+2\beta-1)-2c_m(\beta c_r-c_m)+2\beta(\beta-c_m c_r)]+(\beta-1-c_m+c_r)^2+(1-\beta)(\alpha c_r-4c_m)^2}{16(1-\beta)(2\alpha-8\beta+1)^2}$	$\frac{(\alpha\beta+\alpha-4\beta)^2[5(1-\beta^2)-4(c_m-c_r)(2c_r-2c_m+3\beta-3)]+(\alpha\beta+\alpha-4\beta)[2(\beta-1)^2(\alpha c_m+\alpha c_r+2\beta+2)-4(\beta-1)(\alpha c_m^2-\alpha c_r^2-+4\beta c_m-3\beta c_r+c_m-2c_r)+4(c_m-c_r)(5\beta c_m-3\beta c_r-c_m-c_r)]+(\beta-1)^2(\alpha^2c_m^2+2\alpha^2c_mc_r+\alpha^2c_r^2-4\alpha c_mc_r-2c_mc_r)+(\beta-1)[(\beta+1)^2(\beta-2c_m+2c_r-1)-4\alpha(2\beta c_m^2-\beta c_r^2-c_m^2)+c_m^2(9\beta-5)]+8\beta^2c_m^2-16\beta^2c_mc_r+5\beta^2c_r^2+2\beta c_r^2+c_r^2}{16(1-\beta)(2\alpha\beta+2\alpha-7\beta+1)^2}$
Π_m	$\frac{(\beta-c_m)[\beta(\beta-c_m)-2(\beta c_r-c_m)]-\beta(1-c_r)^2}{8\beta(\beta-1)}$	$\frac{(\beta-c_m)^2[2\alpha^2(\beta-1)-(6\alpha\beta-4\alpha-5\beta+2)]+(2\alpha-3)[2(\beta-c_m)\cdot(\beta c_r-c_m)+\beta(1-c_r)^2]}{8\beta(\beta-1)(3-2\alpha)}$	$\frac{(\alpha-4\beta)^2(\beta-1)+2(\alpha-4\beta)[(c_m-1)(c_m+2\beta-1)-c_r(\beta-1)(\alpha-3)-2c_r(c_m-c_r)+\beta(\beta-c_r^2)]+(1-c_r)(c_r+2\beta-1)+(\beta-1)(\alpha c_r-3c_m)(\alpha c_r-5c_m+2c_r-2)+\beta(c_m^2-2c_mc_r+\beta)}{8(1-\beta)(2\alpha-8\beta+1)}$	$\frac{\beta(\beta-1)(\beta-c_m-c_r+1)\alpha^2-2(3\beta+1)\cdot(-\beta c_r^2+(-2\beta^2+2\beta c_m+2\beta)c_r+\beta^3-2\beta^2c_m+\beta c_m^2+2\beta c_m-2c_m^2-\beta)\alpha+\beta(1-13\beta)c_r^2+2\beta(-11\beta^2+11\beta c_m+12\beta+c_m-1)c_r+9\beta^4-18\beta^3c_m+9\beta^2c_m^2-\beta^3+24\beta^2c_m-23\beta c_m^2-9\beta^2-6\beta c_m+2c_m^2+\beta}{8\beta(1-\beta)(2\alpha\beta+2\alpha-7\beta+1)}$

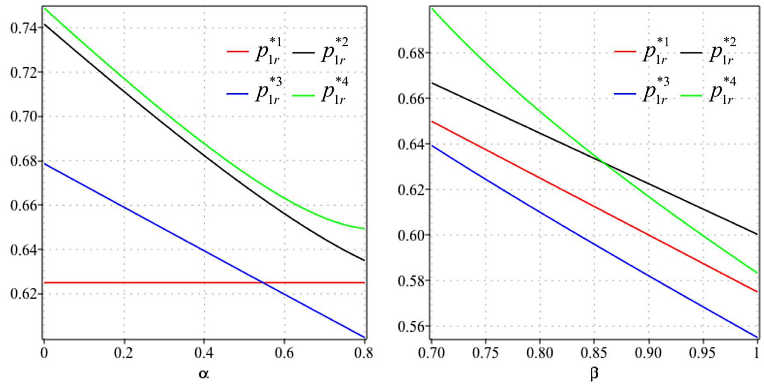

FIGURE 2. Effects on the first-period retail prices under four cases.

5.1.1. Effects on the first-period prices

This part mainly focuses on the impacts of α and β on the first-period retail prices, the direct prices and wholesale prices under four cases. First, we conduct sensitivity analysis of α and β on the first-period retail prices, and the results are shown in Figure 2. It is noting that since all consumers choose to buy the product in the first period, the consumer's patience level will not affect the optimal result of Case 1. Correspondingly, the discussion of the impact of consumers' patience level on Case 1 is not provided in the following analysis. From Figure 2, we find that the first-period retail prices under all cases decrease as the consumer's patience level or the consumer acceptance of the direct channel increases. On the one hand, when consumers become more patient, the decrease of the first-period retail prices can encourage consumers to purchase the product in advance, rather than wait to purchase in the markdown period. When consumers increasingly like the direct channel, the retailer strategically reduces her retail price to attract consumers to purchase from the retail channel. Compared with the first-period retail prices under all cases, we find that the maximum value is under Case 4 when β is less than 0.86 while the maximum value is under Case 2 otherwise. Therefore, when the consumer acceptance of a direct channel can be strong enough, the retailer will set the highest first-period retail price under Case 2. This is ascribed to the following fact that there is no intertemporal competition in the retail channel under Case 2. Thus, the retailer under Case 2 sets a higher retail price in the first period without affecting the sales in the retail channel.

Next, the impacts on the first-period direct prices under four cases are shown in Figure 3. We observe that the impact of the consumer's patience level on the first-period direct prices of all cases are the same as those on the first-period retail prices. The implication is similar to the above analysis for Figure 2. In the following, we analyze how the consumer acceptance of the direct channel affects the first-period direct prices of all cases. Figure 3 shows that the first-period direct prices under all cases increase with the consumer acceptance of the direct channel. It is intuitive that the manufacturer has an incentive to rise his direct price to achieve a high revenue when the direct channel is becoming more popular. On the other hand, as the consumer acceptance of the direct channel increases, the maximum value of the first-period direct price changes from the scenario with markdowns in both channels to the scenario with markdown in the direct channel only, and the increase rate of the latter is the fastest. In other words, the markdown strategy introduced in the retail channel leads the manufacturer to reduce the direct price in the second period. In order to attract consumers to buy the product early, the manufacturer also sets a lower direct price in the first period even though the consumer acceptance level of the direct channel is high enough.

Finally, Figure 4 presents the impacts of α and β on the first-period wholesale prices. We observe that the consumer's patience level has a negative effect on the first-period wholesale prices under all cases, whereas the consumer acceptance of the direct channel affects the first-period wholesale prices differently under different

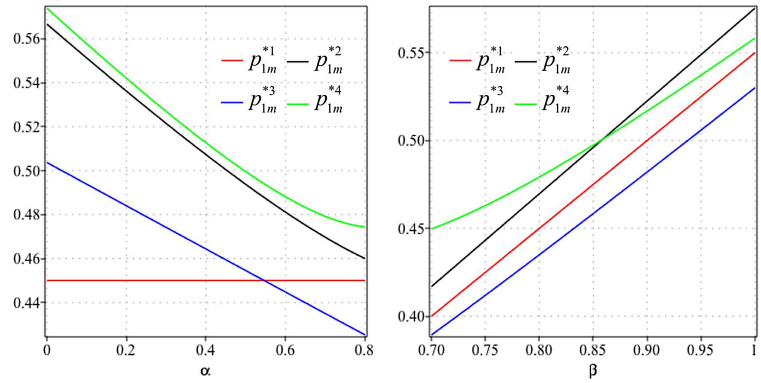

Figure 3. Effects on the first-period direct prices under four cases.

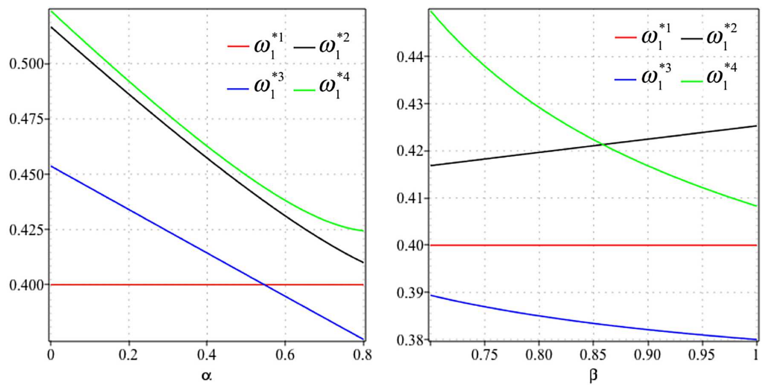

Figure 4. Effects on the first-period wholesale prices under four cases.

cases. On the one hand, when the manufacturer knows that the consumers become more patient, it is good for the manufacturer to reduce the charge of the retailer in the first period, which allows the retailer to lower retail price to stimulate the sales in the retail channel and then improves the order quantities. On the other hand, the second plot of Figure 4 shows that the patterns of the wholesale prices become quite different under all cases. Under Case 2, the first-period wholesale price increases with the consumer acceptance of the direct channel. This main reason is that the manufacturer increases the wholesale price to indirectly reduce the demand of the retail channel, and then to enhance the demand of the direct channel. Under Cases 3 and 4, the first-period wholesale prices decrease with the consumer acceptance of the direct channel. Under Case 3, the manufacturer's profit comes from the retail channel only. Hence, the decrease of the wholesale price can stimulate the retailer to order more. However, under Case 4, based on the action of the first-period retail price, the manufacturer will set a lower wholesale price in the first period to assure the retailer's profit margin and then increase the order quantities from the retail channel.

5.1.2. Effects on the second-period prices

This subsection explores how the consumer's patience level and the consumer acceptance of the direct channel affect the second-period prices, which are presented in Figure 5.

The results show that each of second-period prices under different cases increases with the consumer's patience level. For example, under Cases 2 and 4, the increase in the second-period direct prices for the manufacturer

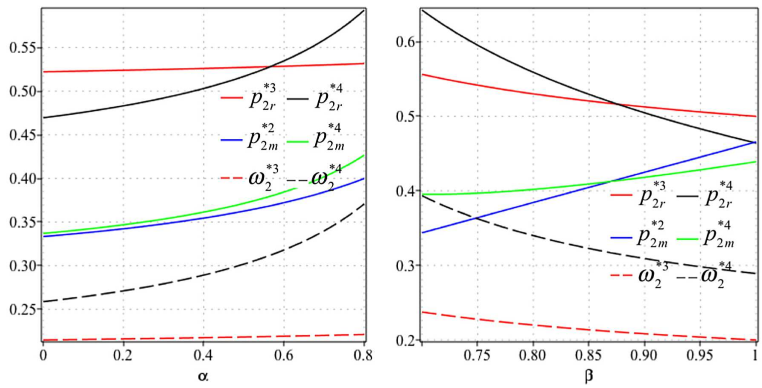

FIGURE 5. Effects of the second-period prices under four cases.

would mitigate the consumers' waiting behavior as the consumers become more strategic. In a similar manner, the retailer also decides to increase her retail prices under Cases 3 and 4 to ensure the sales in the first period. The manufacturer responds by increasing his wholesale prices without reducing the retailer's profit margin in the second period. On the other hand, when the direct channel is increasingly accepted by consumers, the increase in the direct prices under Cases 2 and 4 not only enhances the second-period profit of the manufacturer but also stimulates more consumers to buy the product in the first period. Different from the direct prices, the retail prices under Cases 3 and 4 decrease with the consumer acceptance of direct channel, which is consistent with our intuitions. Correspondingly, the manufacturer lowers his wholesale price in order to assure the second-period profit margin of the retailer. In addition, we observe that the second-period direct price under Case 4 is lower than that under Case 2 when the consumer acceptance of direct channel is sufficiently high. It happens mainly because the markdown adopted by the retailer intensifies the channel competition in the second period, which incurs the manufacturer under Case 4 to set a lower direct price to attract more consumers, even though the consumer acceptance of direct channel is close to that the retail channel. Similarly, the retail price under Case 4 is lower than that under Case 3 if the consumer acceptance of direct channel is sufficiently high. Moreover, the manufacturer under Case 4 always sets a higher second-period wholesale price to ensure the second-period sales of direct channel.

In addition, we further discuss their effects on markdown rate, which is defined as $M_r^k = \frac{p_{1r}^k - p_{2r}^k}{p_{1r}^k}$ in the retail price, $M_m^k = \frac{p_{1m}^k - p_{2m}^k}{p_{1m}^k}$ in the direct price and $M_\omega^k = \frac{\omega_1^k - \omega_2^k}{\omega_1^k}$ in the wholesale price. Figure 6 shows that the markdown rates for all cases decrease with the consumer's patience level and increase with the consumer acceptance of direct channel, except the markdown rate in retail price under Case 3. The reason for the former is that the decrease of markdown rate can prevent the consumers who become more strategic from delaying purchases to certain extent. The reason for the latter is that when the consumer acceptance of the direct channel increase, it is intuitive that the manufacturer increases the markdown rates of direct price and wholesale price so as to enhance sales in the direct channel. However, the retailer decreases the markdown rate of retail channel under Case 3, and increases the discount of retail channel under Case 4. This happens mainly because there is no channel competition under Case 3, the decrease in the markdown rate of retail price makes consumers purchase in advance. The retailer under Case 4 increases the markdown rate, which is resulted from the increase in the discount of wholesale price. In addition, we find that under Case 4, the markdown rate of the wholesale price is the highest, followed by the direct price and the retail price. It indicates that the retail channel offers consumers less discount than the direct channel under markdowns in both channels. The main reason is that the manufacturer advertises the high discount to capture the second-period market share more easily owning to the low operation cost.

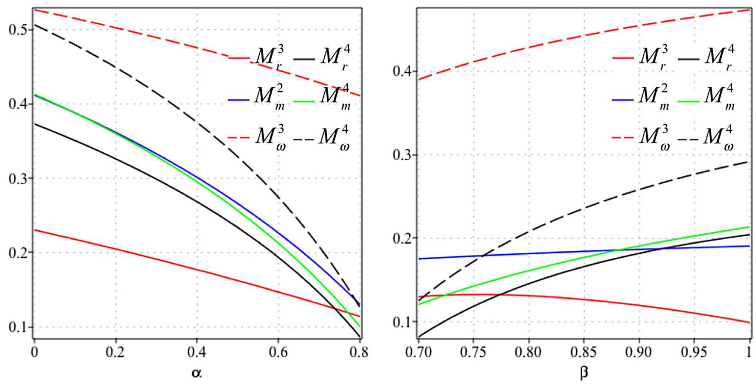

FIGURE 6. Effects of the markdown rates under four cases.

5.2. Effects on profits

In the following, we investigate how the consumer's patience level and the consumer acceptance of the direct channel influence the profits of the retailer, the manufacturer and the whole dual-channel supply chain under four cases.

Figure 7 shows that under all cases, the retailer's profit increases with the consumer's patience level, while it first decreases and then increases with the consumer acceptance of the direct channel. It is noting that the retailer's profit in Case 2 is equal to that in Case 1 (*i.e.*, $\Pi_r^{*1} = \Pi_r^{*2}$) because there is no profit for the retailer in the markdown period under Case 2. The interesting and somewhat surprising results from Figure 7 suggest that the retailer can benefit from consumer strategic behavior despite the slight increase. It happens mainly because with consumers being more strategic, the decrease in the first-period wholesale price and increase in the second-period wholesale price allow the retailer to reduce the first-period retail price and rise the second-period retail price respectively, which entices more consumers to purchase products in the first period and enhance the sales in the first period. On the other hand, the consumer acceptance of the direct channel has a negative effect on the profit of the retailer. This is explained by the fact that the increase in consumer preference for the direct channel dominates the decline of the sales in the retail channel, even though the retail prices in two period are somewhat decreased. However, when the consumer acceptance of the direct channel is close to one, there is little difference between the retail channel and the direct channel. The profit of the retailer increases significantly because the retail channel has advantages over the direct channel, such as inspecting the products in person and enjoying products immediately.

Compared with all cases, the profit of the retailer in Case 4 is the highest when the consumer acceptance of the direct channel is high enough; otherwise the retailer obtains the highest profit from Case 3. This implies that markdown pricing has a positive effect on the retailer in the dual-channel supply chain, as well as that the retailer should reduce prices simultaneously or before the manufacturer. Markdown in the retail channel only illustrates that there is no channel competition in the markdown period. The retailer has the whole market in the markdown period and is able to capture all revenue from markdown market. Hence, the best choice for the retailer is a markdown in the retail channel only. As the differences between both channels become smaller enough, the lower retail prices in two periods stimulate consumers to buy from the retail channel. In addition, the advantage of the retail channel can also attract more consumers, and even attract the consumers of the direct channel. Thus, the retailer achieves rapid profit growth under the case with markdowns in both channels to reach the highest profit.

Figure 8 shows that the profit of the manufacturer decreases with the consumer's patience level, but increases with the consumer acceptance of the direct channel. This occurs mainly due to the following fact. When consumers become more patient, strategic consumers are more inclined to wait for a price discount in the second

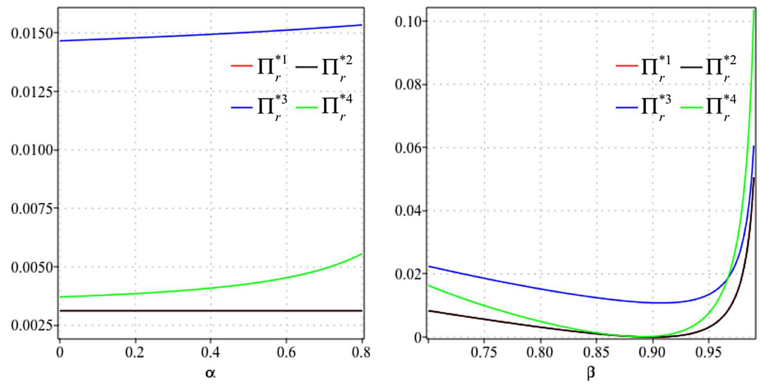

FIGURE 7. Effects on the retailer's profits under four cases.

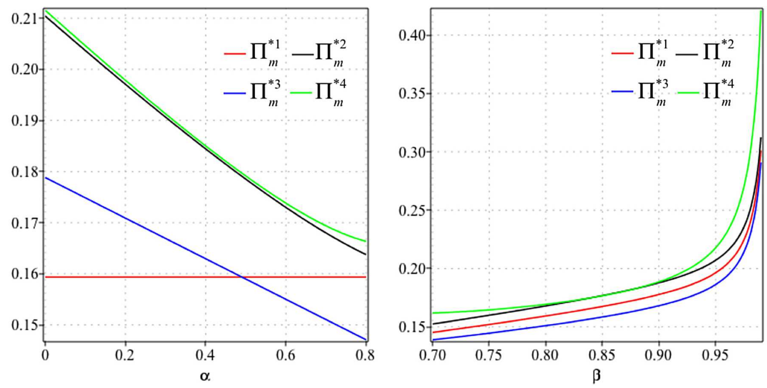

FIGURE 8. Effects on the manufacturer's profits under four cases.

period instead of early purchase at a higher price in the first periods, and the sales in the first period will be reduced. Under such the situation, on one hand, the manufacturer lowers the first-period direct price and rises the second-period direct price to stimulate consumers to buy the product early in the direct channel; on the other hand, the manufacturer also charges a lower wholesale price in the first period to enhance the order quantities. Combining these two aspects, in order to alleviate the strategic behavior of consumers, the lower direct price and wholesale price lead to the reduction of the manufacturer's profit. On the other hand, it is undoubted that the consumer acceptance of the direct channel positively affects the manufacturer's profit. Since the impacts of the consumer's patience level and the consumer acceptance of the direct channel on the profit of the manufacturer are greater than those on the profit of the retailer, their impacts on the whole profit of the dual-channel are the same as that on the manufacturer's profit, as shown in Figure 9.

From a comparison among four cases, the profit of the manufacturer in Case 4 is the highest. For the manufacturer, if the retailer does not adopt a markdown, the manufacturer occupies the whole market share in the second period and obtains an additional revenue from the second period. If the retailer adopts the markdown price, the manufacturer gets the revenue from both the own channel and the retail channel. Hence, markdowns in both the direct channel and the retail channel are the most beneficial for the manufacturer than the other three cases. In comparison, some literatures indicate that the markdown pricing is prone to consumers' strategic waiting behavior that harms the benefits in a traditional supply chain. However, our result reveals that markdown pricing adopted in both channels can bring benefit to the manufacturer under a dual-channel

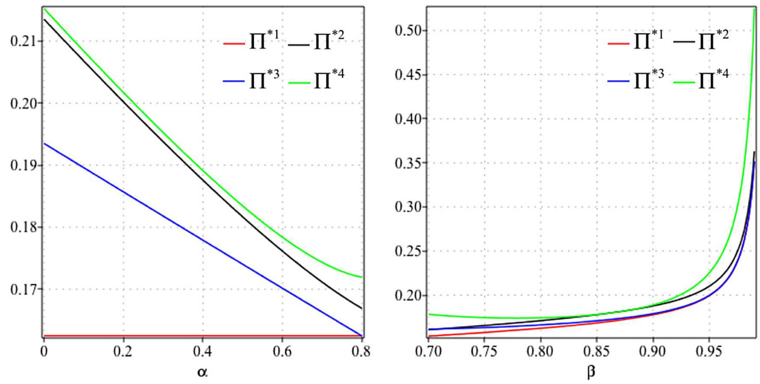

FIGURE 9. Effects on the supply chain's profits under four cases, and $\Pi^{*k} = \Pi_r^{*k} + \Pi_m^{*k}$ $(k = 1, 2, 3, 4)$.

supply chain structure. Finally, we find that the highest profit of the dual-channel supply chain is in Case 4, which results from the fact that profit of the manufacturer is much higher than the profit of the retailer.

6. Conclusions

In this study, we consider a two-period dual-channel supply chain where a manufacturer sells a product *via* both its direct channel and an independent retail channel to strategic consumers in two periods. The manufacturer and the retailer choose whether to adopt a markdown pricing in the direct channel and the retail channel, respectively. Hence, there are four markdown pricing scenarios, including no markdown in both channels, markdown in the direct channel only, markdown in the retail channel only, and markdowns in both channels. For each possible scenario, strategic consumers decide whether, when, and from which channels to purchase the product based on their maximal utilities. By dynamically analyzing consumer purchase behavior, we obtain the demand and the profit functions of the manufacturer and retailer. Then, the optimal pricing decisions of the two players in two periods are derived by solving a two-period game. Finally, we investigate and analyze the effect of the consumer's patience level and the consumer acceptance of the direct channel on the optimal prices and profits. Some key management insights are summarized as follows.

First, under all cases, the consumer's patience level has a negative effect on the first-period pricing decisions but a positive effect on the second-period pricing decisions. In other words, when consumers become more patient, the decrease in the first-period prices and the increase in the second-period prices can induce the consumers to purchase the product in advance rather than wait to purchase in the markdown period. We also observe that the retailer's profit increases and the manufacturer's profit decreases with the consumer's patience level, suggesting that increased consumer's strategic behavior is beneficial for the retailer but harmful for the manufacturer. Since the impact of the consumer's patience level on the manufacturer's profit is greater than on the retailer's profit, the whole profit of the dual-channel supply chain decreases with consumer becoming more patient. Second, with an increase in the consumer acceptance of the direct channel, the retail prices in two periods in all cases decrease while the direct prices increase. Meanwhile, the manufacturer's profit and whole profit of dual-channel supply chain increase with the consumer acceptance of the direct channel, whereas the retailer's profit first decreases followed by an increase. When the consumers increasingly like the direct channel, the manufacturer has an incentive to rise his direct price to achieve a high revenue, which increases the manufacturer's profit. Due to the competition between the direct and retail channels, the retailer strategically reduces her retail price, and then her profit will drop. However, if the consumer acceptance of the direct channel becomes high enough, namely, almost no difference between the retail channel and the direct channel, the retailer's profit increases significantly as a result of that consumers prefer to product from the retail channel rather than the direct channel.

Finally, compared with the four cases, we find that the manufacturer benefits most from the scenario with markdowns in both channels, where the markdown rate of the retail channel is lower than that of the direct channel. The retailer may also profit the most from the scenario with markdowns in both channels when the consumer acceptance of the direct channel is sufficiently high; otherwise the retailer enjoys the highest profit under the markdown only in the retail channel. This suggests that the markdown pricing can bring profitability to the retailer and the manufacturer. On the other hand, compared with the case with markdowns in both channels, the discount offered to consumers by retailers is lower under the case with the markdown only in the retail channel unless consumers are sufficiently strategic or the consumer acceptance level of direct channel is low, whereas the discount offered to retailer by the manufacturer is always higher; meanwhile, the manufacturer offers consumers a lower discount in the case with the markdown only in the direct channel when the consumer's patience level is sufficiently low or the consumer acceptance level of direct channel is sufficiently high.

Due to the limitations presented in the current study, future research could be explored in several directions. First, Stackelberg game strategy is widely applied in operation research [12]. This paper considers a manufacturer-led Stackelberg game in a dual-channel supply chain with two periods, where the manufacturer acts as the leader and the retailer acts as the follower in each period. It is interesting to explore how the manufacturer and the retailer maximize their profits under a retailer-led Stackelberg game model. Second, as manufacturers may use different contracts, such as revenue sharing contract, a two-part tariff contract and so on, to coordinate the total supply chain, the current study uses a simple wholesale price contract commonly applied in practice. The future work by considering how the contracts coordinate the dual-channel supply chain with two periods would provide additional insights. Lastly, we assume in this paper that all information is common knowledge and can be observed by the manufacturer and the retailer. In practice, retailers in direct contact with consumers may withhold true information from manufacturers, resulting in supply chain inefficiencies. Recently, the development of blockchain technology provides a new solution for addressing the key problem of information sharing [11]. Therefore, the impact of blockchain technology on the dual-channel supply chain will be left for future research.

Appendix A. Proof of Theorem 1

By using the backward induction approach, we can obtain Π_r^1 is concave function of p_{1r} since $\frac{\partial^2 \Pi_r^1}{\partial p_{1r}^2} = \frac{2}{\beta-1} < 0$. Hence, the first-order condition $\frac{\partial \Pi_r^1}{\partial p_{1r}} = 0$ gives the optimal retail price as $p_{1r} = \frac{1-\beta+p_{1m}+\omega_1+c_r}{2}$. According to the reaction function of the retailer, the manufacturer maximizes his profit with (ω_1, p_{1m}). Here, Hessian matrix is negative definite since $\frac{\partial^2 \Pi_m^1}{\partial p_{1m}^2} = \frac{2-\beta}{\beta(\beta-1)} < 0$, $\frac{\partial^2 \Pi_m^1}{\partial \omega_1^2} = \frac{1}{\beta-1} < 0$, and $\begin{vmatrix} \frac{\partial^2 \Pi_m^1}{\partial p_{1m}^2} & \frac{\partial^2 \Pi_m^1}{\partial p_{1m} \partial \omega_1} \\ \frac{\partial^2 \Pi_m^1}{\partial p_{1m} \partial \omega_1} & \frac{\partial^2 \Pi_m^1}{\partial \omega_1^2} \end{vmatrix} = \frac{2}{\beta(1-\beta)} > 0$. This shows that there exists a unique equilibrium in the wholesale price and the direct price. By solving the first order necessary conditions for the manufacturer's profit function (see (4.2)), that is $\frac{\partial \Pi_m^1}{\partial \omega_1} = 0$ and $\frac{\partial \Pi_m^1}{\partial p_{1m}} = 0$, the optimal pricing decisions in the first period are given in Theorem 4.1.

Appendix B. Proof of Theorem 2

We can obtain the optimal retail price $p_{1r} = \frac{1-\beta+p_{1m}+\omega_1+c_r}{2}$ by solving the first order condition $\frac{\partial \Pi_r^2}{\partial p_{1r}} = 0$. Substituting p_{1r} into manufacturer's profit function (see (4.5)) yields the profit is jointly concave in (ω_1, p_{1m}) since Hessian matrix is negative definite, that is, $\frac{\partial^2 \Pi_m^2}{\partial p_{1m}^2} = \frac{(\alpha^2\beta+6-4\alpha-2\beta)}{\beta(\beta-1)(2-\alpha)^2} < 0$, $\frac{\partial^2 \Pi_m^2}{\partial \omega_1^2} = \frac{1}{\beta-1} < 0$, and

$\begin{vmatrix} \frac{\partial^2 \Pi_m^2}{\partial p_{1m}^2} & \frac{\partial^2 \Pi_m^2}{\partial p_{1m}\partial \omega_1} \\ \frac{\partial^2 \Pi_m^2}{\partial p_{1m}\partial \omega_1} & \frac{\partial^2 \Pi_m^2}{\partial \omega_1^2} \end{vmatrix} = \frac{2(2\alpha-3)}{\beta(\beta-1)(2-\alpha)^2} > 0$. This shows that there exists a unique equilibrium in wholesale price and direct price by solving $\frac{\partial \Pi_m^2}{\partial \omega_1} = 0$ and $\frac{\partial \Pi_m^2}{\partial p_{1m}} = 0$, as shown in Theorem 4.2.

Appendix C. Proof of Theorem 3

In a similar manner, there exists a unique equilibrium in retail price $p_{1r} = \frac{1-\beta+p_{1m}+\omega_1+c_r}{2}$ by solving $\frac{\partial \Pi_r^3}{\partial p_{1r}} = 0$. Substituting p_{1r} into manufacturer's profit Π_m (see (4.9)), Hessian matrix is given as follows: $\frac{\partial^2 \Pi_m^3}{\partial p_{1m}^2} = \frac{\alpha^2-16\beta^2-8\alpha+36\beta-4}{(\beta-1)(\alpha-4\beta)^2}$, $\frac{\partial^2 \Pi_m^3}{\partial \omega_1^2} = \frac{1}{\beta-1} < 0$ and $\begin{vmatrix} \frac{\partial^2 \Pi_m^3}{\partial p_{1m}^2} & \frac{\partial^2 \Pi_m^3}{\partial p_{1m}\partial \omega_1} \\ \frac{\partial^2 \Pi_m^3}{\partial p_{1m}\partial \omega_1} & \frac{\partial^2 \Pi_m^3}{\partial \omega_1^2} \end{vmatrix} = \frac{4(-8\beta+2\alpha+1)}{(\beta-1)(2-\alpha)^2}$. To assure that Hessian matrix is negative definite, $A(\beta) = -16\beta^2 + 36\beta + \alpha^2 - 8\alpha - 4 > 0$ and $B(\beta) = -8\beta + 2\alpha + 1 < 0$ must hold. That is to say, $\frac{9-\sqrt{4\alpha^2-32\alpha+65}}{8} < \beta < \frac{9+\sqrt{4\alpha^2-32\alpha+65}}{8}$ and $\beta > \frac{1+2\alpha}{8}$. According to the following fact that $\frac{9-\sqrt{4\alpha^2-32\alpha+65}}{8} < \frac{1+2\alpha}{8} < 1 < \frac{9+\sqrt{4\alpha^2-32\alpha+65}}{8}$, we can obtain that when $B(\beta) < 0$, $A(\beta) > 0$ always hold. Therefore, based on the assumption $\beta > \alpha$, we find when $\beta > \max\{\frac{1+2\alpha}{8}, \alpha\}$, the Hessian matrix is negative definite, which means that Π_m^3 is jointly concave in (ω_1, p_{1m}). By solving $\frac{\partial \Pi_m^3}{\partial \omega_1} = 0$ and $\frac{\partial \Pi_m^3}{\partial p_{1m}} = 0$, the optimal pricing decisions for two players in the first period are presented in Theorem 4.3.

Appendix D. Proof of Lemma 1

From the retailer's profit, we can obtain Π_{2r}^4 is concave function of p_{2r} since $\frac{\partial^2 \Pi_{2r}^4}{\partial p_{2r}^2} = \frac{2}{\beta-1} < 0$. Hence, the first-order condition $\frac{\partial \Pi_{2r}^4}{\partial p_{2r}} = 0$ gives the unique retail price as $p_{2r} = \frac{\theta(1-\beta)+p_{2m}+\omega_2+c_r}{2}$. Substituting p_{2r} into manufacturer's profit (4.11), the profit is jointly concave in (ω_2, p_{2m}) since $\frac{\partial^2 \Pi_{2m}^4}{\partial p_{2m}^2} = \frac{2-\beta}{\beta(\beta-1)} < 0$, $\frac{\partial^2 \Pi_{2m}^4}{\partial \omega_2^2} = \frac{1}{\beta-1} < 0$, $\begin{vmatrix} \frac{\partial^2 \Pi_{2m}^4}{\partial p_{2m}^2} & \frac{\partial^2 \Pi_{2m}^4}{\partial p_{2m}\partial \omega_2} \\ \frac{\partial^2 \Pi_{2m}^4}{\partial p_{2m}\partial \omega_2} & \frac{\partial^2 \Pi_{2m}^4}{\partial \omega_2^2} \end{vmatrix} = \frac{2}{\beta(1-\beta)} > 0$. Thus, the optimal wholesale price and the direct price are given in Lemma 4.4 by solving $\frac{\partial \Pi_{2m}^4}{\partial \omega_2} = 0$ and $\frac{\partial \Pi_{2m}^4}{\partial p_{2m}} = 0$.

Appendix E. Proof of Theorem 4

There exists a unique equilibrium in retail price $p_{1r} = \frac{1-\beta+p_{1m}+\omega_1+c_r}{2}$ by solving $\frac{\partial \Pi_r^4}{\partial p_{1r}} = 0$. Substituting p_{1r} into the manufacturer's profit (4.13) gets Hessian matrix as follows: $\frac{\partial^2 \Pi_m^4}{\partial p_{1m}^2} = \frac{(\alpha^2-12)\beta^2+(2\alpha^2-8\alpha+32)\beta+\alpha^2-8\alpha-4}{(\beta-1)(\alpha\beta+\alpha-4\beta)}$, $\frac{\partial^2 \Pi_m^4}{\partial \omega_1^2} = \frac{1}{\beta-1} < 0$ and $\begin{vmatrix} \frac{\partial^2 \Pi_m^4}{\partial p_{1m}^2} & \frac{\partial^2 \Pi_m^4}{\partial p_{1m}\partial \omega_1} \\ \frac{\partial^2 \Pi_m^4}{\partial p_{1m}\partial \omega_1} & \frac{\partial^2 \Pi_m^4}{\partial \omega_1^2} \end{vmatrix} = \frac{4(2\alpha\beta+2\alpha-7\beta+1)}{(\beta-1)(\alpha\beta+\alpha-4\beta)^2}$. Similar to Theorem 3, we can obtain when $\beta > \max\{\frac{1+2\alpha}{7-2\alpha}, \alpha\}$, $\frac{\partial^2 \Pi_m^4}{\partial p_{1m}^2} < 0$ and $2\alpha\beta + 2\alpha - 7\beta + 1 < 0$ hold; then, the Hessian matrix is negative definite. As a result, the manufacturer's profit is jointly concave in (ω_1, p_{1m}). By solving simultaneously $\frac{\partial \Pi_m^4}{\partial \omega_1} = 0$ and $\frac{\partial \Pi_m^4}{\partial p_{1m}} = 0$, the optimal wholesale price and the direct price are given in Theorem 4.5.

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