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A neural network approach to control performance assessment

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Abstract

Purpose – The purpose of this paper is to present a neural network approach to control performance assessment.

Design/methodology/approach – The performance index under study is based on the minimum variance control benchmark, a radial basis function network (RBFN) is used as the pre-whitening filter to estimate the white noise sequence, and a stable filtering and correlation analysis method is adopted to calculate the performance index by estimating innovations sequence using the RBFN pre-whitening filter. The new approach is compared with the auto-regressive moving average model and the Laguerre model methods, for both linear and nonlinear cases.

Findings – Simulation results show that the RBFN approach works satisfactorily for both linear and nonlinear examples. In particular, the proposed scheme shows merits in assessing controller performance for nonlinear systems and surpasses the Laguerre model method in parameter selection.

Originality/value – A RBFN approach is proposed for control performance assessment. This new approach, in comparison with some well-known methods, provides satisfactory performance and potentials for both linear and nonlinear cases.

Keywords Control technology, Performance appraisal, Analysis of variance, Correlation analysis

Paper type Research paper

1. Introduction

In a typical process industry facility, lots of control loops often make it difficult and time-consuming to keep all of them operating satisfactorily. Many factors can contribute to the poor performance of control loops, such as inadequate controller or inappropriate control structure, equipment malfunction, and unmeasured disturbances change. Therefore, it is necessary to find out an important tool for control engineers to detect which control loops need to be paid attention to, and also necessary to find out which of them causes this poor performance and diagnose the underlying problem.

The performance of an existing control loop is often measured against some types of benchmarks, such as offset from setpoint, overshoot, rise time, and variance. For regulatory control, the variance of output is an important performance measure since many process and quality release criteria are based on variance. The key point is that the minimum variance (MV) benchmark (as a reference performance bound) can be estimated from routine operating data without additional experiments, provided the system delay d is known (or can be estimated with sufficient accuracy). Harris (1989) firstly showed that the theoretical lower bound of closed-loop output variance can be

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estimated from routine closed-loop operating data, and an important feature of the method is that it is not necessary to perturb the routine operation of the process with extraneous test signal. The celebrated MV-based performance index has been suggested by Harris (1989), and thus is also referred to as the Harris index. The underlying principles originate from the work by Åström (1970) and Box and Jenkins (1970), who established the theory of minimum-variance control and DeVries and Wu (1978), who used these ideas for performance assessment. Desborough and Harris (1992) connected the Harris index to the squared correlation coefficients usually calculated in multiple regression analysis. Lynch and Dumont (1996) reported the use of Laguerre network to model the closed-loop system in order to estimate the MV control for controller performance monitoring. Eriksson and Isakson (1994) gave some aspects of control loop performance monitoring for not stochastic control scheme. Huang *et al.* (1997) developed an efficient, stable filtering and correlation (FCOR) analysis method to estimate the MV benchmark. Some other excellent work on control loop performance assessment can be found in reviews by Harris *et al.* (1999), Qin (1998), Huang and Shah (1999), Jelali (2006) and Wei *et al.* (2008).

In recent years, many control performance assessment methods for linear process have been reported; unfortunately, there are few approaches for the nonlinear cases. When extending the methodology of linear system performance assessment to nonlinear systems, still, there are many challenges in model determination and parameters estimation. Chen *et al.* (1990) presented the development of nonlinear MV controllers for processes that admit a nonlinear ARMAX representation. Bittanti and Piroddi (1993) gave a MV control for nonlinear plants with neural networks. Harris and Yu (2007) used Volterra series approximation for estimation of the MV bounds for a class of nonlinear systems.

Following the idea in Harris and Yu (2007) for a class of nonlinear systems with feedback-only schemes, we propose a neural network approach for controller performance assessment. In Section 2, a SISO process description is first given and the controller performance index based on MV control benchmark is illuminated. FCOR principles and different whitening methods including the RBFN are described in Section 3. In Section 4, the proposed approach is tested and compared with several typical methods through numerical simulations for both linear and nonlinear examples. The paper is concluded with some future considerations in Section 5.

2. Description of process and performance assessment index

2.1 Description of SISO linear process and MV control benchmark

In what follows, a SISO process under regulatory control as shown in Figure 1 is considered, where y_t is the process output, u_t is the process input, a_t is a white noise with zero mean and constant variance σ_a^2, d is the time delay, G_u is the delay-free plant

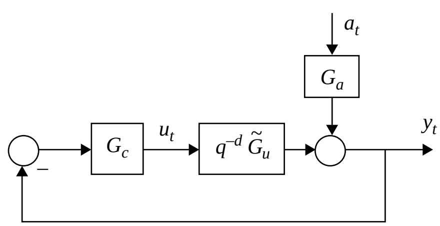

Figure 1.
SISO process regulation with feedback-only schemes

transfer function, G_a is the disturbance transfer function, and can be represented by auto-regressive moving average (ARMA) time series model, G_c is the controller transfer function. The process model can be written as:

$$y_t = G_u u_t + G_a a_t, \tag{1}$$

where G_u is the control channel transfer function, which usually has the following form:

$$G_u = q^{-d} G_u. \tag{2}$$

For any discrete system, d is always larger than 1. Such a decomposition of G_u will help us identify how much process noise we are not able to eliminate. This part of noise is directly related to MV, and what is more important, this MV is invariant to controller design.

When a linear time invariant feedback controller is used to regulate the output around a fixed zero setpoint by G_c:

$$u_t = -G_c y_t. \tag{3}$$

Then in Figure 1, the closed-loop output y_t turns to be:

$$y_t = \frac{G_a}{1 + q^{-d} G_c G_u} a_t. \tag{4}$$

Decomposing G_a into two parts using Diophantine identity gives:

$$G_a = f_0 + f_1 q^{-1} + \cdots + f_{d-1} q^{-d+1} + R q^{-d} = F + R q^{-d}. \tag{5}$$

Then equation (4) can be written as:

$$y_t = \frac{G_a}{1 + q^{-d} G_c G_u} a_t = F a_t + \frac{R - F G_c G_u}{1 + q^{-d} G_c G_u} q^{-d} a_t = F a_t + L a_t, \tag{6}$$

where f_i are impulse response constant coefficients, R is the remaining rational proper transfer function, and $F a_t = (f_0 + f_1 q^{-1} + \cdots + f_{d-1} q^{-d+1}) a_t$ is the portion of MV control output independent of feedback control, L is a proper transfer function.

When the MV control benchmark is used, and since the two terms on the right hand side of equation (6) are independent, the variance of the output can be expressed as:

$$\mathrm{var}(y_t) = \mathrm{var}(F a_t) + \mathrm{var}(L a_{t-d}) \geq \mathrm{var}(F a_t). \tag{7}$$

The equality holds if $L = 0$, i.e.:

$$R - F G_c G_u = 0, \tag{8}$$

which yields the MV control law:

$$G_c = \frac{R}{F G_u}. \tag{9}$$

2.2 MV performance bounds and feedback invariants for nonlinear systems

In linear systems, the effect of process disturbances can always be correctly represented as an output disturbance regardless of where they actually appear in the system, this is a consequence of the principle of superposition. But for nonlinear systems, superposition does not hold. When facing nonlinear systems that are superposition of a nonlinear process model plus a linear stochastic disturbance model, it is useful however to provide an additive description. For discrete models, a most general form of process description looks like as in Harris and Yu (2007):

$$y_t = f_p\left(u^*_{t-d}, w^*_t\right) + D_t. \tag{10}$$

The expressions and symbols have the same meaning as what in Harris and Yu (2007), where D_t is the additive distance which can be represented by a linear ARMA model. The derivation of MV controller for a process described by equation (10) is straightforward, as in Grimble (2002). Therefore, equation (10) can be written as:

$$\begin{aligned} y_{t+d} &= f_p\left(u^*_t, w^*_{t+d}\right) + D_{t+d} = f_p\left(u^*_t, w^*_{t+d}\right) + \hat{D}_{(t+d)/t} + e_{(t+d)/t} \\ &= \hat{y}_{(t+d)/t} + e_{(t+d)/t}. \end{aligned} \tag{11}$$

If it is possible to find the control action at time t such that $f_p(u^*_t, w^*_{t+d}) + \hat{D}_{(t+d)/t} = 0$, then the resulting controller is the MV controller. $e_{(t+d)/t}$ is the feedback invariant MV performance bounds, in the form of:

$$e_{(t+d)/t} = (1 + f_1 q^{-1} + \cdots + f_{d-1} q^{-(d-1)}) a_{t+d}, \tag{12}$$

where weights f_i are the impulse coefficients of the closed-loop transfer function. Therefore, the MV or the invariant portion of output variance, which is the lower bound on performance, as measured in mean square sense, is:

$$\sigma^2_{\mathrm{mv}} = \mathrm{var}\left\{y^{\mathrm{mv}}_{t+d}\right\} = \left(f_0^2 + f_1^2 + \cdots + f_{d-1}^2\right)\sigma^2_a. \tag{13}$$

In order to measure control performance, the following performance index is used:

$$\eta(d) = \frac{\sigma^2_{\mathrm{mv}}}{\sigma^2_y}. \tag{14}$$

The σ^2_y term in equation (14) can be calculated from the routine closed-loop operating data. Consequently, this σ^2_{mv} term should be estimated from routine operating data.

Generally speaking, there are two ways for estimating the MV or the feedback invariant portion:

(1) *Estimation of closed-loop impulse response*. In this approach, a time series model given is fit to closed-loop data. The first $d-1$ impulse coefficients are estimates of the first $d-1$ coefficients of the open-loop disturbance transfer function. With the estimated coefficients and an estimate of σ^2_a obtained from the model-estimation stage, the MV performance can be estimated. An autoregressive moving average (ARMA) model can be used for estimating the

impulse response transfer function. Laguerre network model can also be used to estimate the impulse response transfer function between the output and the white noise input, as well as the estimation of the white noise. In FCOR algorithm, FCOR method is used to estimate impulse response coefficients, the details are given in the next section.

(2) *Direct estimation from the routine operating data.* Desborough and Harris (1992) adopted a lagged regression of the form:

$$y_{t+d} = \hat{y}_{t+d} + e_{t+d} = \xi(q^{-1})y_t + e_{t+d}. \tag{15}$$

This can be estimated from routine closed-loop operating data. The residual variance from the model fitting provides an estimate of the MV performance.

For nonlinear processes, this paper uses a radial basis function network (RBFN) as the nonlinear predictor for estimating the MV or the feedback invariant portion. The MV performance bounds can be directly estimated from a representative sample of closed-loop data when the process is adequately modeled and $\hat{y}_{(t+d)/t}$ can be accurately constructed. The RBFN for estimation of the MV bounds is given in the next section.

3. FCOR algorithm and methods for whitening filter

Huang *et al.* (1997) developed an efficient, stable FCOR method to estimate the MV benchmark. The key point is that the MV benchmark (as a reference performance bound) can be estimated from routine operating data without additional experiments, provided the system delay d is known (or can be estimated with sufficient accuracy). The pre-whitening step is equivalent to finding a suitable time-series for whitening filter, for instance, AR or ARMA models that can be used for estimating the white noise sequence. Whitening is actually to reconstruct or estimate the white noise $\hat{a}_t$ (of course, $\hat{a}_t$ is no long equal to the real white noise on the process). The identification of innovation models has attracted much interest.

3.1 Description of FCOR algorithm

A stable closed-loop process can be written as an infinite-order moving average process, and the impulse response parameters for a closed-loop system may be expressed as:

$$y_t = H(t)a_t = (f_0 + f_1 q^{-1} + \cdots + f_{d-1}q^{-d+1} + f_d q^{-d} + \cdots)a_t, \tag{16}$$

where $H(t)$ is the impulse transfer function between y_t and a_t, and f_i is its coefficients.

When the performance index in equation (14) is used, and after some statistic correlation analysis, the corresponding sampled estimation of performance index is therefore written as:

$$\hat{\eta}(d) = \hat{\rho}^2_{ya}(0) + \hat{\rho}^2_{ya}(1) + \cdots + \hat{\rho}^2_{ya}(d-1). \tag{17}$$

The sampled estimation given by:

$$\hat{\rho}_{ya}(k) = \frac{\frac{1}{L}\sum_{t=1}^{L} y_t a_{t-k}}{\sqrt{\frac{1}{L}\sum_{t=1}^{L} y_t^2 \frac{1}{L}\sum_{t=1}^{L} a_t^2}}, \tag{18}$$

where ρ_{ya} is the cross-correlation coefficient between y_t and a_t for lag 0 to $d-1$, L is the sample length. Although a_t is unknown in equation (18), it can be replaced by the estimated innovations sequence $\hat{a}_t$. The estimated $\hat{a}_t$ is obtained by pre-whitening the process output variable y_t via time series analysis. The process of obtaining such a "whitening" filter is analogous to time series modeling, where the final test of the adequacy of the model consists of checking if the residuals are "white," where these residuals are the estimated white noise sequence.

3.2 White noise filter or pre-whitening

3.2.1 AR type model based on adaptive whitening filter. The estimated $\hat{a}_t$ is obtained by pre-whitening the process output variable y_t via time series analysis, and the estimation of this noise sequence is important for performance assessment. The coefficients f_i of the impulse response from noise-to-output transfer function have to be estimated, for instance, using an ARMA model. When the deadtime is small, it means that only few data points are required to fit a full ARMA or continuous model. If the order of the noise model is assumed to be small (such as an AR(1) or AR(2) models), then the parameters in these models may be directly estimated from an impulse response of the closed-loop system.

Here, it is assumed that the disturbance is stable and therefore can be represented by a finite amount of parameters. Equation (16) is unsuitable for parameter estimation as it depends on the unknown sequence $\hat{a}_t$. However, for a discrete form, it may be transformed into an equivalent sequence:

$$y_k = (\beta_1 q^{-1} + \beta_2 q^{-2} + \cdots + \beta_m q^{-m})y_k + a_k = \varphi(k-1)^{\mathrm{T}}\theta + a_k, \tag{19}$$

where $\varphi(k) = [y_k, y_{k-1}, \ldots, y_{k-m}]^{\mathrm{T}}$, $\theta = [\beta_1, \beta_2, \ldots, \beta_m]^{\mathrm{T}}$ collects the parameters to be identified, and m is the number of repressor variables.

In this section, the Wiener filter-based MV control scheme is extended as a direct adaptive MV self-tuning regulator. The controller consists of two-parameter adaptation algorithms (PAA) running simultaneously. The PAA is an adaptive whitening filter that identifies the parameters $\hat{\beta}_i$, and produces an estimate of the innovation signal a_t. Given the parameter estimate and regressor vectors of the adaptive whitening filter $\hat{\theta} = [\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_m]^{\mathrm{T}}$, the parameters $\hat{\beta}_i$ may be fit by performing a recursive least squares (RLS) PAA algorithm based on the closed-loop data $y_t, y_{t-1}, \ldots, y_{t-m}$. Define the predicted output as:

$$\hat{y}(k) = \varphi(k-1)^{\mathrm{T}}\hat{\theta}(k-1), \tag{20}$$

then the a priori estimation error of the adaptive whitening filter looks like:

$$e(k) = y(k) - \hat{y}(k) = y(k) - \varphi(k-1)^{\mathrm{T}}\hat{\theta}(k-1), \tag{21}$$

and the RLS algorithm follows like:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\varphi(k-1)e(k), \tag{22}$$

$$p(k) = p(k-1) - \frac{p(k-1)\varphi(k-1)\varphi(k-1)^{\mathrm{T}}p(k-1)}{\varphi(k-1)^{\mathrm{T}}p(k-1)\varphi(k-1)}. \tag{23}$$

The a priori error $e(k)$ will be used as an estimate of the innovation signal a_k in the overall adaptive scheme. Once β_i are estimated, the impulse weight parameters may easily be determined from the formula (Hugo, 2006):

$$\hat{f}_i = \sum_{j=1}^{i} \beta_j \hat{f}_{i-j} (f_0 = 1), \quad i = 1, 2, \ldots, d-1. \tag{24}$$

Rather, when the sample length is large enough, the closed-loop impulse response can be identified from the cross correlation function of the residuals $e(k)$ with the output $y(k)$ as in Tyler and Morari (1996):

$$\hat{f}_i = \frac{1}{n-i} \sum_{j=i}^{n} e(j-i) y(j). \tag{25}$$

With the help of equations (24) or (25), the estimated MV turns to be:

$$\sigma_{\mathrm{mv}}^2 = (\hat{f}_0^2 + \hat{f}_1^2 + \cdots + \hat{f}_{d-1}^2) \sigma_e^2. \tag{26}$$

3.2.2 Laguerre network model. Laguerre network model can be used to estimate the impulse response transfer function between the output and the white noise input, and the estimation of the white noise are also given. The use of an ARMA model to estimate impulse response transfer function means that the degrees of the numerator and denominator of the model have to be determined. Although techniques are available to facilitate this choice, it is not trivial as the filter can be very complicated and a poor choice may cause the estimation to suffer. The use of the Laguerre network is becoming more common, due to its attractive properties. The discrete Laguerre filters can be written as:

$$L_i(q) = \frac{\sqrt{1-a^2}}{q-a} \left(\frac{1-aq}{q-a} \right)^{i-1}, \quad i = 1, 2, \ldots, \tag{27}$$

where a is the time scale of the filter. As the Laguerre functions are orthonormal and complete in $L_2[0, \infty)$, a stable impulse transfer function can be approximated as:

$$H(q^{-1}) = \sum_{i=1}^{N} g_i L_i(q^{-1}), \tag{28}$$

where N is the truncated constant of Laguerre network and g_i are the Laguerre gains.

Once the filter time scale and the number of filters are set, the Laguerre gains that best approximate impulse transfer function need to be determined. For this, it is convenient to represent the discrete Laguerre network in state-space form:

$$\mathbf{L}(k+1) = A\mathbf{L}(k) + Ba(k), \quad y(k) = \mathbf{C}^{\mathrm{T}}\mathbf{L}(k) + a(k), \tag{29}$$

where the $\mathbf{C}$ and $\mathbf{L}$ vectors are, respectively, defined as:

$$\mathbf{C} = [g_1, g_2, \ldots, g_N]^{\mathrm{T}}, \quad \mathbf{L} = [l_1, l_2, \ldots, l_N]^{\mathrm{T}}. \tag{30}$$

After given the terms A and B, which depend only on the filter time scale and the number of filters are set, the minimum achievable output variance can be determined using equation (13) and is:

$$\sigma_{\mathrm{mv}}^{2} = (1 + (\mathbf{C}^{\mathrm{T}}B)^{2} + (\mathbf{C}^{\mathrm{T}}AB)^{2} + \cdots + (\mathbf{C}^{\mathrm{T}}A^{d-2}B)^{2})\sigma_{a}^{2}, \tag{31}$$

where $\mathbf{C}^{\mathrm{T}}B, \mathbf{C}^{\mathrm{T}}AB, \ldots, \mathbf{C}^{\mathrm{T}}A^{d-2}B$ are the Markov parameters of the process. Upon close examination of equation (29), it can be seen that the input to the Laguerre network is the unknown white noise sequence $a(k)$. To estimate the gains, the input must thus also be estimated. To perform this, the recursive extended least squares (LS) estimation is used:

$$\mathbf{L}(k+1) = A\mathbf{L}(k) + Bv(k-1), \tag{32}$$

$$p(k) = p(k-1) - \frac{p(k-1)\mathbf{L}(k)\mathbf{L}(k)^{\mathrm{T}}p(k-1)}{1 + \mathbf{L}(k)^{\mathrm{T}}p(k-1)\mathbf{L}(k)}, \tag{33}$$

$$\hat{\mathbf{C}}(k) = \hat{\mathbf{C}}(k-1) + P(k)\mathbf{L}(k)[y(k) - \hat{\mathbf{C}}(k-1)^{\mathrm{T}}\mathbf{L}(k)], \tag{34}$$

$$v(k) = y(k) - \hat{\mathbf{C}}(k)^{\mathrm{T}}\mathbf{L}(k). \tag{35}$$

The residual $v(k)$ gives an estimate of the white noise $a(k)$, and $\hat{\sigma}_{v}^{2}$ can be used to estimate σ_{a}^{2}, which is required in equation (31). The estimated MV will then be:

$$\hat{\sigma}_{\mathrm{mv}}^{2} = (1 + (\hat{\mathbf{C}}^{\mathrm{T}}B)^{2} + (\hat{\mathbf{C}}^{\mathrm{T}}AB)^{2} + \cdots + (\hat{\mathbf{C}}^{\mathrm{T}}A^{d-2}B)^{2})\hat{\sigma}_{v}^{2}. \tag{36}$$

3.2.3 RBF neural network structure. A viable alternative to highly nonlinear-in-the-parameter neural networks is the RBFN. Here, we choose the radial basis function centers one-by-one in a rational way until an adequate network has been constructed, based on orthogonal least squares (OLS) learning algorithm. The significant regressors can be selected in this forward regression manner. An RBF network can be regarded as a special two-layer network which is linear in the parameters by fixing all RBF centers and nonlinearities in the hidden layer. Therefore, the hidden layer performs a fixed nonlinear transformation with no adjustable parameters and it maps the input space onto a new space. The output layer then implements a linear combiner on this new space and the only adjustable parameters are the weights of this linear combiner. These parameters can therefore be determined using the linear LS method, which gives an important advantage of this approach. Chen *et al.* (1989) used OLS method as a forward regression procedure to select a suitable set of centers (regressors) from a large set of candidates. At each step of the regression, the increment to the explained variance of the desired output is maximized. Furthermore, oversize and ill-conditioning problems occurring frequently in random selection of centers can automatically be avoided. This rational approach provides an efficient learning algorithm for fitting adequate RBF networks.

A schematic of the RBF network with n inputs and a scalar output is shown in Figure 2. Such a network implements a mapping $f : R^{n} \rightarrow R$ according to:

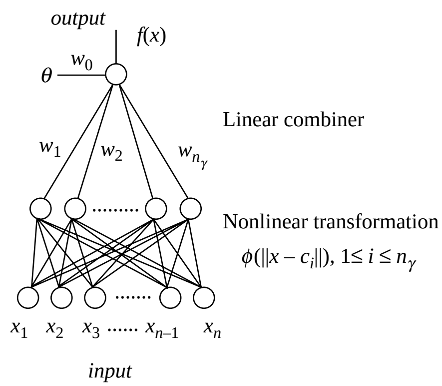

Figure 2.
Schematic of RBFN

$$f(x) = w_0\theta + \sum_{i=1}^{n_r} w_i \phi(\|x - c_i\|). \tag{37}$$

Alternatively, the OLS algorithm can be used to select centers so that adequate and parsimonious RBF networks can be obtained. In order to understand how this works, it is essential to view the RBF network equation (37) as a special case of the linear regression model:

$$d(k) = \sum_{i=1}^{M} w_i p_i(k) + a(k), \tag{38}$$

where $d(k)$ is the desired output, w_i are the parameters to be estimated and $p_i(k)$ are known as the regressors as Gaussian functions of x:

$$p_i(k) = \exp\left(-\frac{|x - c_i|^2}{\sigma_i^2}\right). \tag{39}$$

After estimating the weight w_i, the estimation of white noise sequence can be given by equation (37), where the input consists of $y_k, y_{k-1}, \ldots, y_{k-n}$.

In the following, the proposed RBFN approach is used for estimating the MV or the feedback invariant portion for nonlinear systems.

4. Numerical simulation and algorithm comparison

4.1 Comparison among different approaches for linear systems

In order to test the proposed RBFN approach and compare with other filter structures based on FCOR algorithm for performance assessment, the following frequently-used SISO process with time delay $d = 2$ is considered (Desborough and Harris, 1992; Huang and Shah, 1999):

$$y_t = u_{t-2} + \frac{1 - 0.2q^{-1}}{1 - q^{-1}} a_t, \tag{40}$$

where a_t is normally distributed, with mean 0 and standard deviation 0.36. When a simple integral feedback controller is chosen:

$$\Delta u_t = -K y_t, \tag{41}$$

where K is the integral gain, it can be shown that the closed-loop response is given by:

$$y_t = a_t + 0.8a_{t-1} + \frac{0.8(1 - (K/0.8) - Kq^{-1})}{1 - q^{-1} + Kq^{-2}} a_{t-2} = \frac{1 - 0.2q^{-1}}{1 - q^{-1} + Kq^{-2}} a_t, \tag{42}$$

where the first two terms $a_t + 0.8a_{t-1}$ compose the MV portion, which is independent on the feedback controller. For comparison, the following control performance assessment methods are investigated:

- The general approach proposed by Harris (1989) (denoted as the ARMA approach). This approach uses an ARMA model for estimating the closed-loop transfer function between y_t and a_t, and an adaptive whitening filter for training parameter and getting the estimation of the white noise sequence. The performance index can be acquired after polynomial long division or solving the Diophantine equation of the transfer function.
- The Laguerre network method. Lynch and Dumont (1996) proposed to use Laguerre network to model the process system, which can eliminate the need to solve the Diophantine equation in finding the MV.
- An efficient, stable FCOR method developed by Huang *et al.* (1997) is also used for control performance assessment. The pre-whitening step is equivalent to finding a suitable time-series for whitening filter. The ARMA model based on adaptive filter, the Laguerre network model and the proposed RBFN can be used for whitening filter from the routine operating data, provided the system delay d is known (or can be estimated with sufficient accuracy).

To perform the simulation, a series of standard normal random variants are generated using the method in Park and Miller (1988). These random variants are tested for independence using statistic analysis methods. The series is not used if the hypothesis that all the autocorrelations from lag 1 to lag 10 are zero is rejected at the 95 percent confidence level. Once an acceptable a_t series is generated, the process output can then be calculated using equation (42). The true value of MV lower bound is 0.5904 and the estimated MV bounds are calculated as the residual variance from each model, and this procedure is repeated five hundred times with different white noise sequences. About 1,000 realizations of equation (42) as shown in Figure 3 are generated in each procedure with normal distribution white noise with zero 0 and variance 0.36.

After an iterative procedure of model order selection and parameter estimation for the ARMA model, an AR(6) model is used to estimate the MV lower bound by using equation (13). The time scale and filter number of Laguerre network model are given as 0.2 and 6, respectively, and the estimated MV lower bound can be obtained from equation (36). When the OLS is used for training RBFN, six significant regressors are selected in a forward regression manner.

The comparison results for $K = 0.5$ are shown in Table I, where the listed values are the means of 500 simulations calculated for each model. From the data in Table I, the following observations can be made:

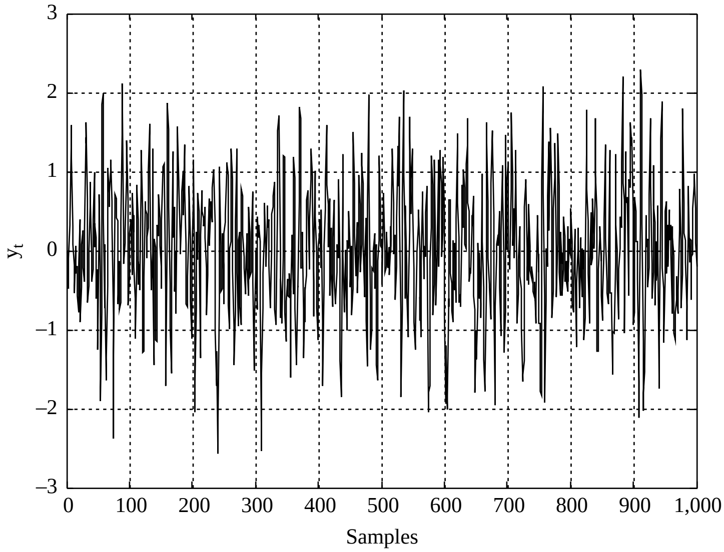

Figure 3.
The data of the closed-loop output with $k = 0.5$

- When using AR models to estimate the closed-loop transfer function, the degree of polynomials, which has big impact on the estimation of σ^2_{mv}, must be chosen properly. Then the performance index can be calculated after taking polynomial long division or solving the Diophantine equation.
- The Laguerre network method and all the FCOR-based algorithms can estimate the MV lower bound through the estimation of closed-loop impulse response, through which both can eliminate the need of solving the Diophantine equation.
- The proposed RBFN approach based on the FCOR algorithm provides a better accuracy than the Laguerre network model.
- All the FCOR-based algorithms give smaller estimated MVs than the direct estimations of the MV lower bound. However, the filter and correlation analysis of routine closed-loop operating data gives the estimation of closed-loop impulse response, thus provides a useful insight into control loop performance analysis.

For the RBFN approach, Figure 4 shows the estimated control performance versus the theoretical performance for different controller integral gain K. As the FCOR algorithm

	$\hat{\sigma}^2_{mv}$	$\sigma^2_{\hat{a}}$	$\hat{\eta}(d)$
ARMA	0.5890	0.3592	0.9146
L-network	0.5886	0.3573	0.9140
FCOR-based methods			
ARMA	0.5485	0.3592	0.8517
L-network	0.5668	0.3573	0.8801
RBFN	0.5774	0.3585	0.8966

Table I.
Estimates of $\eta(d)$ using different models ($\sigma^2_y = 0.6440$)

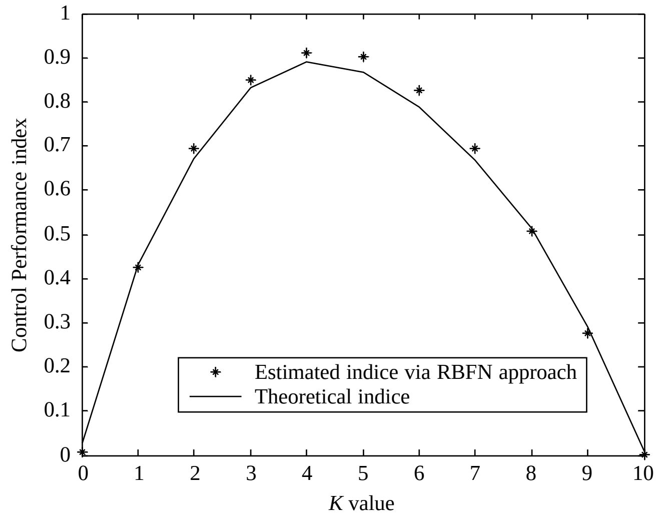

Figure 4. RBFN approach: the estimated and theoretical control performance indices at different *K* values

assumes that the white noise is really "white" (that satisfies normal distribution with zero mean), higher estimations of the performance index occur at some points.

4.2 Extension to nonlinear systems

Although at any time an invariant linear system can be completely characterized by its impulse response, or equivalently by an autoregressive model, unfortunately, this equivalence cannot be extended to all nonlinear problems. For a class of nonlinear systems, the development of nonlinear MV controllers has been reported by Chen *et al.* (1990) for processes that admit a nonlinear ARMAX representation, and an estimation of lower bound from operating data using Volterra series approximation has been given in Harris and Yu (2007).

In the following, an example is provided to demonstrate the methodology outlined in this paper. Consider the nonlinear dynamic system represented by a second order Volterra series (Harris and Yu, 2007):

$$y_t = 0.2u_{t-2} + 0.3u_{t-4} + u_{t-5} + 0.8u_{t-3}^2 + 0.8u_{t-3}u_{t-4} - 0.7u_{t-4}^2 - 0.5u_{t-5}^2 - 0.5u_{t-3}u_{t-5} + \tilde{D}_t. \tag{43}$$

The disturbance is an ARIMA (2, 0, 0) process:

$$D_t = \frac{a_t}{1 - 1.6q^{-1} + 0.8q^{-2}}, \tag{44}$$

where a_t is a white noise sequence with zero mean and variance 0.1. The true value of the MV lower bound is 0.6656. Assuming the setpoint equals to zero, then a proportional controller can be used to control the simulated process:

$$u_t = -0.2 y_t. \quad (45)$$

For estimating the MV lower bounds, three direct estimation methods are compared:

(1) Linear autoregressive (LAR) model: $y_{t+d} = \sum_{i=0}^{m} b_i y_{t-i}$. For a linear model, it is also convenient to fit the data using an ARMA representation.

(2) Laguerre network model is used to fit the nonlinear process and the estimation of the white noise can be used for the FCOR algorithm to examine the approach efficiency.

(3) The proposed RBFN approach based on FCOR algorithm is tested to assess the control performance for the nonlinear process.

In this simulation, 500 observations are used to fit the parameters for these models. When formulating the models, a large number of candidate terms are initially allowed. About 1,000 realizations of equation (43) are used to estimate the MV lower bound. The results for the proportional (P) controller assessment are shown in Table II.

4.3 Residual analysis

When the FCOR algorithm is used for performance assessment, the whitening filter and correlation analysis are important. It is necessary to check if the residuals are "white," where these residuals are the estimated white noise sequence.

Suppose a_t is a white noise sequence, and let $a(1), a(2), \ldots, a(L)$ be some sample values, where L is the sample length. Then we find correlation among the residuals themselves. $\rho_a(k)$ is the autocorrelation coefficients of a_t for lag 0 to $d-1$, defined as:

$$\rho_a(k) = \frac{R_a(k)}{R_a(0)}, \quad (46)$$

where $R_a(k)$ is the autocorrelation function of a_t, and $\rho_a(k)$ can be estimated from finite length samples:

$$\hat{\rho}_a(k) = \frac{\hat{R}_a(k)}{R_a(0)}, \quad (47)$$

where:

$$\hat{R}_a(k) = \frac{1}{L} \sum_{t=1}^{L-1} a_t a_{t+k}. \quad (48)$$

When L is large enough, under the assumption that a_t are white noise sequence, the L statistical value $\sqrt{L}\hat{\rho}_a(1), \sqrt{L}\hat{\rho}_a(2), \ldots, \sqrt{L}\hat{\rho}_a(m)$ satisfy normal distribution $N(0, 1)$, and the square sum of them should be asymptotically $\chi^2(m)$ distributed:

	$\hat{\sigma}^2_{mv}$	$\hat{\eta}(d)$
ARMA	0.6842	0.4392
RBFN	0.6751	0.4334
Laguerre-network	0.6668	0.4281

Table II. Estimates of $\eta(d)$ using different models ($\sigma_y^2 = 1.5577$)

$$T = \sum_{k=1}^{m} \left[\sqrt{L} \hat{\rho}_a(k) \right]^2 = L \sum_{k=1}^{m} \hat{\rho}_a(k). \tag{49}$$

For checking if the residuals are "white," the sequence whiteness test can through testing whether T satisfies $\chi^2(m)$ distributed. If $T \leq \chi^2_\alpha(m)$, the α level of the $\chi^2(m)$ distribution, the white sequence will pass a test of being $\chi^2(m)$ distribution, otherwise a_t is not a white sequence, and the whitening filter model need be modified to get a new "white" sequence. In the simulation part, we choose $m = 30$, $\alpha = 0.05, L = 1,000$, and then we have $\chi^2_{0.05}(30) = 18.49$.

For the proposed RBFN based on FCOR algorithm for the control performance assessment, to give an accurate whitening identification model, the residual analysis is necessary. The autocorrelation function of the estimated white noise sequence with the 95 percent confidence level appears in Figure 5, from which we see that the estimated white noise sequence is white as the autocorrelation at all lags lies near zero, and with a variance of 0.6751. In this residual analysis, we get $T = 15.24 < 18.49$, which indicates T satisfies $\chi^2(m)$ distribution.

4.4 Discussions

From the simulation comparisons, it can be seen that the proposed RBFN and the Laguerre network methods perform better than the ARMA model approach for both linear and nonlinear examples.

It seems that no obvious overall advantage can be seen of using the proposed RBFN approach. We would comment, however, it took a significant amount of experimental work to determine where to truncate the polynomial expansion in equation (19) for the ARMA model structure or how to choose the parameters like the time scale for the Laguerre network model. For a given plant, the truncation error of a Laguerre network model using a truncated Laguerre series to represent plant dynamics will be a function of the number of filters and their time scale. For a fixed number of filters, the optimum time scale that minimizes the truncation error depends on characteristics of the system impulse response. To obtain a fast rate of convergence, the time scale should be chosen close to the dominating time constants of the plant to be approximated while this is usually not an easy task. Numerical procedures including exhaustive search

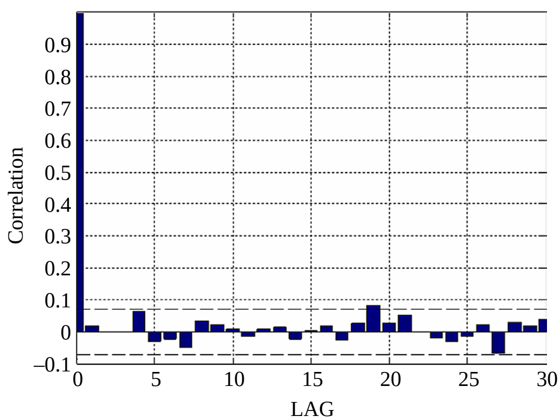

Figure 5.
Autocorrelation function of the estimated white noise sequence

algorithms, the nonlinear search simplex method and the Fibonacci search approach can be used, yet the result of these search methods depend on the Laguerre filters number used, and need large amount of calculation.

The proposed RBFN is in essence used to identify the unknown noise sequence. This is quite equivalent to standard system modeling or identification via neural network approach which is normally fast enough for real-time applications. RBFN has been proved as a powerful model structure and widely used in system modeling and identification practice. The parameters c_i and w_i of the proposed RBFN construction are, however, not difficult to select by using efficient algorithms like the OLS learning method. On the other hand, usually it is assumed that the plant, the controller and the disturbance change little during a short assessment process. Under this circumstance, the assessment can be deemed as an offline process and thus the proposed RBFN approach has no difficulty in providing timely assessments.

5. Conclusions and future work

A RBFN approach has been proposed for control performance assessment. Two simple but frequently-used examples are given to illustrate how the new approach operates and compares with some well-known methods. It is found that the proposed RBFN approach works satisfactorily for both linear and nonlinear examples. In particular, it provides additional useful information that the underlying plant seems to be adequately represented by this neural network model and thus shows capability in assessing controller performance for nonlinear plants. Moreover, the RBFN approach surpasses the Laguerre network method in parameter selection. The OLS method greatly facilitates the updating of the parameters that enter the RBFN in a nonlinear fashion.

On the other hand, there are still many challenges in control performance assessment for nonlinear systems. This paper presents only a preliminary attempt of using neural network to the problem. Some immediate future work includes applying the proposed approach to some practical control systems and choosing different neural network models for identification.

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