STAGNATION-SHORTENING INEXACT NEWTON METHOD BASED ON UNSUPERVISED LEARNING FOR HIGHLY NONLINEAR HYPERELASTICITY PROBLEMS ON THREE-DIMENSIONAL UNSTRUCTURED MESHES

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Abstract. Inexact Newton method is widely used in many scientific and engineering applications, but in some situations its convergence involves a long stagnation period in which the nonlinear residual doesn't decrease much and a large percentage of the total compute time is wasted. The reasons for the stagnation are quite difficult to quantify. In this paper, we propose a stagnation-shortening inexact Newton algorithm with a stagnation analysis phase during the Newton iterations in which the norms of the residuals are used as a guiding curve and the vector values of the residuals are centralized and decomposed into a slow subspace and a regular subspace using an unsupervised learning method based on singular value decomposition. A targeted quasi-Newton is then performed in the slow space whose solution is later projected back to the global space. We show numerically that with such an embedded learning phase the inexact Newton method converges almost quadratically. As an application, we consider the modeling of the human artery with stenosis using the hyperelasticity equation with multiple material parameters. Due to the significant difference in the material coefficients between the plaques and the healthy parts of the blood vessels, the problem is nonlinearly very difficult. Numerical experiments demonstrate that proposed method offers significantly reduced number of nonlinear iterations and robustness for this rather tough hyperelasticity problem.

Key words. Inexact Newton, learning-based nonlinear preconditioning, singular value decomposition, hyperelasticity, finite element, parallel computing

1. Introduction.

Inexact Newton-type methods are widely used for solving large system of algebraic equations arising from the discretization of nonlinear partial differential equations for modeling scientific and engineering problems in, for examples, aerospace engineering, material sciences, and computational medicine, etc. [3, 4, 10, 11]. When successful, the methods take only a small number of iterations to reach the solution due to the quadratic convergence rate, however, for nonlinearly ill-conditioned problems, the methods may converge slowly or not converge at all. Figure 1.1 shows a typical residual curve obtained using an inexact Newton method for a hyperelastic problem of human artery with stenosis. The problem is highly nonlinear when multiple material parameters are involved, and a good initial guess is often absent, leading to a stagnation in the residual curve. The bad nonlinearities can hardly be reduced in the stagnation period by classical globlization techniques, such as linesearch and trust region mehtods. Because of the jump of the material coefficient, and the large deformation, the problem is considered to be highly nonlinear and the convergence of inexact Newton is often problematic, as shown in Figure 1.1. The long stagnation period consumes a lot of compute time without making much progress in reducing the nonlinear residual. Note that for such a highly nonlinear problem, the linear Jacobian system that one has to build and solve at each Newton iteration is large and highly ill-conditioned. As shown in the three sub-figures in Figure 1.1, the linear solvers take a large number of Krylov iterations to converge, but their converged solutions (i.e., sometimes called Newton directions) don't help

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