Durable Goods and Nondurable Consumption

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Abstract

We present an integrated framework to analyze durable goods in conjunction with nondurable consumption in consumer demand and consumption by incorporating the defining features of durable goods. We consider relative prices of nondurable goods and the user cost of durable goods allowing for costly reversibility and adjustment costs. Then we examine risk aversion and intertemporal allocation of nondurable consumption and durable goods.

Keywords: User cost, Adjustment costs, Costly reversibility, Indirect utility function, Euler equations

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1. Introduction

Traditionally, studies on consumer behavior are conducted using data on nondurable goods in micro demand analysis (see, e.g., Deaton and Muellbauer, 1980; Banks et at., 1997) and in macro consumption analysis (see, e.g., Hall, 1978; Hansen and Singleton, 1983), tacitly assuming that nondurable goods are separable from durable goods. Durable goods are, however, essential like most nondurable goods and represent an important fraction of consumer wealth; thus ignoring them may not provide an adequate portrayal of consumer behavior.

To analyze durable goods, it is important to account for the fact that consumers derive utility by jointly consuming durable and nondurable goods. Therefore, ignoring the interaction between these two types of goods is potentially misleading for a proper understanding of consumer behavior. In this context, there is a growing body of empirical work analyzing consumer behavior with nondurable consumption by incorporating durable goods, especially in analysis of consumption (see, e.g., Mankiw, 1985; Chah et al., 1995; Alessie et al, 1997; Ogaki and Reinhard, 1998; Browning and Crossley. 2009; Pakoš, 2011). These studies are, however, based on highly aggregated data and the use of restrictive utility functions that are additively separable in nondurable consumption and durable goods, which likely leads to a biased inference. In essence, they lack solid theoretical underpinnings of consumer behavior with no due regard to the components of nondurable consumption, and leave unexplained the role of relative prices of nondurable goods in determining consumption.

We provide here an integrated framework to analyze durable goods along with nondurable goods in consumer demand and consumption via intertemporal two-stage budgeting (see Kim et al., 2021) by incorporating the salient features of durable goods. For durable goods, unlike nondurable goods, consumers derive utility from the flow of services they provide over time. This implies that the relevant price of durable goods is not the purchase price, as with nondurable goods, but the cost of using their services, i.e., the user cost of these goods. Furthermore, consumers face costs of adjusting durables stocks. These costs involve costs associated with installation, delivery, maintenance, searching and learning, and disposal, which directly affect consumer behavior about durable goods. Durable goods are also characterized by irreversibility or costly reversibility; that is, purchases of durable goods are impossible or costly to reverse, when the resale price of durable goods may be zero or is below their purchase price. Irreversibility acts as a form of adjustment costs. The presence of a secondhand or secondary market for durable goods admits of partial or costly reversibility and hence mitigates irreversibility of these goods.

There are studies on durable goods analyzing adjustment costs and irreversibility with the (S, s) model (see Bar-Illan and Blinder, 1992; Eberly, 1994). They, however, do not allow for the user cot of durable goods and fail to account for the interplay between durable and nondurable goods, which is the focus of our study. We do not consider these studies in our analysis.

2. Analytical Framework

We consider a representative consumer who faces an optimal consumption problem of nondurable and durable goods over time in the presence of adjustment costs. This problem can be solved in two stages in accordance with intertemporal two-stage budgeting. In the first stage, the levels of nondurable consumption and durables stock are chosen by optimally allocating wealth across periods. Then, in the second stage, each period's optimal allocation of nondurable expenditure is distributed across nondurable goods conditional on durables stock. We assume that there is a secondary market for durable goods, but to focus on reversibility and adjustment costs and thus to facilitate the analysis, we assume that capital markets are perfect. The solution to the above budgeting procedure can be found by reversing the order of the two stages: first, solve the second stage problem and then the first stage problem.

2.1. Indirect Utility Function and Short-run Demands for Nondurable Goods

Let $\mathbf{q}_t$ be an *n* quantity vector of nondurable goods at period *t* and k_t the level of durables stock at the end of period *t*. Given a direct utility function, $u(\mathbf{q}_t, k_t)$, which is continuous, increasing, and quasi-concave in $\mathbf{q}_t$ and k_t, the consumer's second stage optimization problem is summarized by the indirect utility function conditional on durables stock, $v(C_t, \mathbf{p}_t, k_t)$, defined as

$$v(C_t\,, \mathbf{p}_t, k_t) \equiv \max_{\mathbf{q}_t}\{u(\mathbf{q}_t, k_t) \,\big|\, \mathbf{p}_t \cdot \mathbf{q}_t \le C_t\,\}, \tag{1}$$

where C_t is the consumption expenditure to be allocated among nondurable goods at period *t*, and $\mathbf{p}_t$ is an *n* x1 vector of the prices of nondurable goods at period *t*. The indirect utility function is well defined as a description of the consumer's within-period preferences under the following regularity conditions: it is continuous, increasing in C_t and k_t, decreasing in $\mathbf{p}_t$, homogeneous of degree zero in $(C_t,\ \mathbf{p}_t)$, and quasi-convex in $\mathbf{p}_t$ (see Deaton and Muellbauer, 1980). Application of Roy's identity to the conditional indirect utility function (1) yields the system of short-run demand functions for nondurable goods:

$$q_{it} = g_i(C_t, \mathbf{p}_t, k_t) = -\frac{\partial \nu(C_t, \mathbf{p}_t, k_t) / \partial p_{it}}{\partial \nu(C_t, \mathbf{p}_t, k_t) / \partial C_t},\ i = 1, \ldots, n, \tag{2}$$

where $g_i(C_t, \mathbf{p}_t, k_t)$ is the ordinary or Marshallian demand function for the ith $(i = 1, \ldots, n)$ nondurable good conditional on durables stock.

2.2. Intertemporal Optimization with Nondurable Consumption and Durable Goods

The second stage optimization problem, characterized by (1), is derived under the assumption that the consumer takes, as given, nondurable expenditure and durables stock. The first stage problem of intertemporal two-stage budgeting allows us to determine them endogenously in the consumer's intertemporal optimization decision. In particular, durables stock, like physical capital, is quasi-fixed – fixed in the short run but variable in the long run. It evolves over time according to

$$k_s = (1-\delta)k_{s-1} + q_s^k \quad \text{for all } s \geq t, \tag{3}$$

where q_s^k is the quantity of durable goods purchased at period s, and δ is the depreciation rate of durable goods assumed to be constant.

There are costs of adjustment associated with changing durables stock specified by a function $h\left(q_s^k\right)$. For analytical tractability, we take a quadratic (which is convex) function of the form for $h(q_s^k)$: $h\left(q_s^k\right) = \frac{\varphi\left(q_s^k\right)^2}{2}$ for $\varphi > 0$, which gives the marginal adjustment cost $h'\left(q_s^k\right) = \varphi q_s^k$. With this adjustment cost function, the consumer faces an intertemporal finance or budget constraint:

$$A_s = (1 + r_{s-1}) A_{s-1} + Y_s - C_s - p_s^k q_s^k - h\left(q_s^k\right) \quad \text{for all } s \geq t, \tag{4}$$

where A_s is the value of financial assets at the end of period s to be carried into the next period, r_{s-1} is the nominal interest rate on assets that can be both bought and sold at period s-1, Y_s is labor income at period s, and p_s^k is the price of durable goods purchased at period s.

The consumer's first stage optimization problem is to choose C_s and k_s, for all $s \geq t$, so as to maximize

$$E_t\left[\sum_{s=t}^{\infty}(1+\rho)^{-(s-t)}\left(\frac{v(C_s\,,\,\mathbf{p}_s\,,k_s\,)^{1-\zeta}-1}{1-\zeta}\right)\right], \tag{5}$$

where ρ is the rate of the consumer's time preference, subject to the durables stock accumulation equation (3), the intertemporal budget constraint (4), and the appropriate transversality conditions for assets and durables stock, and E_t is the expectation operator taken over future variables, using information available at the beginning of period *t*. We assume that the consumer replans continuously when solving the above stochastic control problem. Then, for estimation and data analysis, only the first-order conditions necessary for the intertemporal optimization problem (5) at the initial point in time (*s*=*t*) are relevant. They are given by

$$C_t\ : v(C_t\,,\,\mathbf{p}_t\,,k_t\,)^{-\zeta}\,v_C(C_t\,,\,\mathbf{p}_t\,,k_t\,)=\mu_t \tag{6a}$$

$$A_t : \mu_t\ = E_t\left[\left(\frac{1+r_{t+}}{1+\rho}\right)\mu_{t+1}\right], \tag{6b}$$

$$q_t^k\ :\left(p_t^k\ + h'(q_s^k)\right)\mu_t\ = \lambda_t^k\,, \tag{7a}$$

and

$$k_t : v(C_t\,,\,\mathbf{p}_t\,,k_t\,)^{-\zeta}\,v_k(C_t\,,\,\mathbf{p}_t\,,k_t\,)=\lambda_t^k - E_t\left[\left(\frac{1-\delta}{1+\rho}\right)\lambda_{t+1}^k\right], \tag{7b}$$

where μ_t is the Lagrange multiplier associated with (4) measuring the marginal utility of wealth, whilst λ_t^k is the Lagrange multiplier related to (3) measuring the shadow price of a unit of installed durable goods.

For empirical analysis, it is convenient to work with the above first-order conditions in a ratio form. From (6a) and (6b), we have

$$E_t\left[\left(\frac{1+r_t}{1+\rho}\right)\frac{\mu_{t+1}}{\mu_t}\right]=E_t\ \left[(1+r_t\ )D_{t+1}\right]=1, \tag{8}$$

where $D_{t+1}\equiv\mu_{t+1}/\mu_t(1+\rho)$, referred to as the stochastic discount factor. Combining (6a), (7a), and (7b), and substituting for $h'(q_s^k)$, we obtain

$$\frac{v_k(C_t\,,\,\mathbf{p}_t\,,k_t\,)}{v_C(C_t\,,\,\mathbf{p}_t\,,k_t\,)}=\left(p_t^k\ -(1-\delta)E_t\ \left[D_{t+1}\,p_{t+1}^k\right]\right)+\varphi\left(q_t^k\ -(1-\delta)E_t\ \left[D_{t+1}q_{t+1}^k\right]\right). \tag{9}$$

2.3. The User Cost of Durable Goods with Adjustment Costs

The user cost of the service flow from durable goods r_t^k at period *t* is defined as (see Deaton and Muellbauer, 1980, Chapter 13):

$$r_t^k = p_t^k - \frac{(1-\delta)}{(1+r_t)} p_{t+1}^k. \tag{10}$$

Given a resale or secondary market for durable goods with no transaction cost, the user cost equals the net expense of buying a unit of durable goods in one period at p_t^k using it in the same period and selling it in the next period at p_{t+1}^k. Assuming that p_t^k grows by $\Delta \ln p_{t+1}^k$ and approximating (10), the user cost of durable goods is usually considered their rental equivalent price, i.e., $r_t^k \approx p_t^k (r_t + \delta - \Delta \ln p_{t+1}^k)$, where $\Delta \ln p_{t+1}^k$ is the expected rate of inflation of durable goods (see below for a further discussion).

The user cost allows for costly or partial reversibility via a secondary market by treating complete irreversibility as a special case. Costly reversibility means that it is easy to purchase durable goods but is costly to reverse the purchase with partial recovery of the purchase price. It is believed that a lack of secondary markets for durable goods causes irreversibility, suggesting that uncertainty about future shocks makes consumers cautious or hesitant to purchase new durable goods (see Knotek and Kahn, 2011). Adjustment costs can make durable goods costly to reverse. In reality, there is a prevalence of secondary or secondhand markets for many durable goods, which reduce the cost associated with irreversibility. To the extent that the resale price of durable goods p_{t+1}^k is less than their purchase price p_t^k, however, irreversibility of durable goods is not completely eliminated, but rather mitigated, in the second hand market, making purchases of durable goods costly, if not impossible, to reverse.

The user cost of durable goods (10) does not account for adjustment costs. We can derive it under adjustment costs from the first-order conditions of the intertemporal optimization problem, particularly from (9). To do so, suppose that D_{t+1} and p_{t+1}^k in (9) are uncorrelated so that $E_t\left[D_{t+1} p_{t+1}^k\right] = E_t\left[D_{t+1}\right]$ x $E_t\left[p_{t+1}^k\right]$. We also assume that D_{t+1} and q_{t+1}^k in (9) are uncorrelated. Assuming further that the asset is risk free, Equation (8) implies that $E_t\left[D_{t+1}\right] = 1/(1+r_t)$. Using these results, Equation (11) can be rewritten as

$$\frac{v_k(C_t\ ,\ \mathbf{p}_t\ ,k_t\ )}{v_C(C_t\ ,\ \mathbf{p}_t\ ,k_t\ )}=\left(p_t^k-\left(\frac{1-\delta}{1+r_t}\right)E_t\ \left[p_{t+1}^k\right]\right)+\varphi\left(q_t^k-\left(\frac{1-\delta}{1+r_t}\right)E_t\ \left[q_{t+1}^k\right]\right),$$

or, approximately as

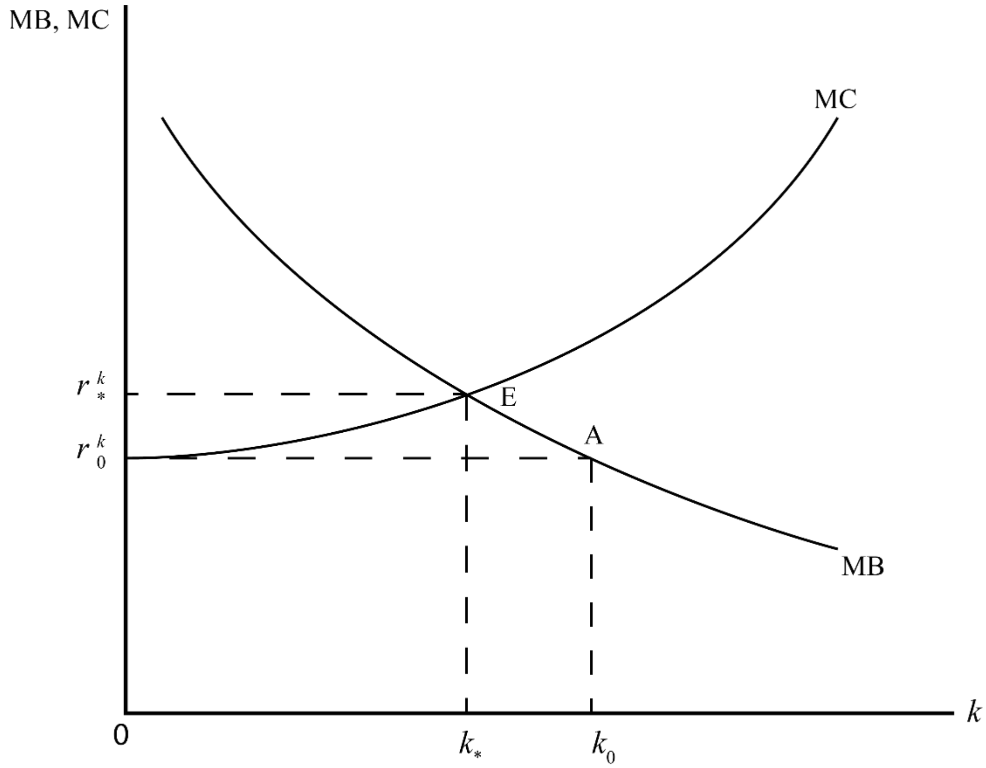
$$\frac{v_k(C_t\ ,\ \mathbf{p}_t\ ,k_t\ )}{v_C(C_t\ ,\ \mathbf{p}_t\ ,k_t\ )}\approx r_t^k+\varphi\left(q_t^k-\left(\frac{1-\delta}{1+r_t}\right)q_{t+1}^k\right). \tag{11}$$

The left-hand side of this expression is the marginal willingness to pay (or MB) for durable services, which is a decreasing function of k_t. On the one hand, the right-hand side of (11) measures the marginal cost (MC) of using durable services, which is equal to the user cost plus the net marginal adjustment cost of buying a unit of durable goods in one period, using it in the same period and selling or disposing it in the next period. Since the marginal adjustment cost of durable goods is increasing in q_t^k, the MC of durable services is an increasing function of q_t^k and hence k_t.

Equation (11) describes an optimal decision rule for durable services in the presence of adjustment costs for a given r_t^k. At the optimum, MB = MC and an optimal level of k_t is determined conditional on a given level of C_t. Note that adjustment costs place a wedge between the user cost and the consumer's marginal willingness to pay for durable services. Due to this wedge, durables stock does not respond immediately to exogenous shocks. This implies that if adjustment costs are not accounted for, the traditional user cost will likely to overstate the consumer's true marginal willingness to pay for durable services, leading to a level of durables stock that is higher than optimal.

The aforesaid results are illustrated in Figure 1. For a given user cost of durable goods r_0^k, the optimal level of durables stock (k_*) is determined at the point E where MB = MC. In the presence of adjustment costs, the consumer no longer plans to choose k_0 at the point where $r_0^k = MB$, which is valid only under costless adjustment of durables stock. The adjustment cost of pushing durables stock toward that level acts as a brake that slows down the optimal pace of adjustment. Since $r_*^k > r_0^k$, this clearly suggests that the failure to allow for adjustment costs results in a higher level of durables stock. Furthermore, a change in the user cost r^k causes a shift in the MC curve of durable goods. It is also worth noting that if adjustment costs are not accounted for, a decline in the user cost overstates the optimal level of k, while a rise in the user cost understates it.

Figure 1: Effect of Adjustment Costs of Durable Services on Durables Demand



Notes: MB=marginal benefit of durable services, MC= marginal cost of durable services, r^k = user cost of durable services, and k = durables stock. Point A represents optimum with no adjustment costs, while point E represents optimum with adjustment costs.

For a given user cost r_t^k, we define the adjusted or generalized user cost of durable goods r_t^{ak} that allows for adjustment costs as

$$r_t^{ak} \equiv r_t^k + \varphi\left(q_t^k - \left(\frac{1-\delta}{1+r_t}\right) q_{t+1}^k \right). \tag{12}$$

Given (11), Equation (12) could be rewritten as

$$r_t^{ak} = \psi(C_t, \mathbf{p}_t, k_t) = \frac{v_k(C_t, \mathbf{p}_t, k_t)}{v_C(C_t, \mathbf{p}_t, k_t)}, \tag{13}$$

where $\psi(C_t, \mathbf{p}_t, k_t)$ is an inverse demand function for durable goods conditional on nondurable consumption. If we express (13) as $v_C(C_t, \mathbf{p}_t, k_t) = v_k(C_t, \mathbf{p}_t, k_t) / r_t^{ak}$ and substitute it into (8) using (6a), it follows that

$$E_t \left[\left(\frac{1+r_t}{1+\rho} \right) \left(\frac{r_t^{ak}}{r_{t+1}^{ak}} \right) \frac{\mu_{t+1}^k}{\mu_t^k} \right] = 1, \tag{14}$$

where $\mu_t^k \equiv v(C_t, \mathbf{p}_t, k_t)^{-\zeta} v_k(C_t, \mathbf{p}_t, k_t),$ which describes the *intertemporal* allocation of durable goods. Equations (8) and (14), which are the Euler equations for nondurable consumption and durable goods expressed in a ratio form, are used in the analysis of the intertemporal model (see subsection 2.6).

2.4. Long-run Demands for Durable and Nondurable Goods

The short-run demands for nondurable goods in (2) are conditional on fixed durables stock, and durables stock in the inverse demand for durable goods (13) is treated as exogenous. However, the consumer can freely adjust durables stock in the long run. Implicitly solving $r_t^{ak} = \psi(C_t, \mathbf{p}_t, k_t)$ in (13) for k_t, we obtain a long-run demand function for durable goods of the form:

$$k_t = k^{LR1}(C_t, \mathbf{p}_t, r_t^{ak}). \tag{15}$$

Substituting (15) into (2) gives the long-run demand functions for nondurable goods:

$$q_{it} = g_i^{LR1}(C_t, \mathbf{p}_t, r_t^{ak}) = g_i \left[C_t, \mathbf{p}_t, k^{LR1}(C_t, \mathbf{p}_t, r_t^{ak}) \right], \; i = 1, \ldots, n. \tag{16}$$

It should be stressed that adjustment costs of durable goods, by increasing the adjusted user cost, affects the demands for durable as well as nondurable goods. With the long-run demands for durable goods (15) and for nondurable goods (16), we may derive and estimate long-run elasticities of these goods conditional on nondurable expenditure. Nonetheless, nondurable consumption is endogenously determined and expected to adjust in response to a change in income or total purchasing power allocated to nondurable and durable consumption. Henceforth, it is more appropriate to measure the long-run elasticities by conditioning them on total consumption expenditure.

Define the total consumption expenditure on durable and non-durable goods as $M_t \equiv \mathbf{p}_t \cdot \mathbf{q}_t + r_t^{ak} k_t = C_t + r_t^{ak} k_t,$ which gives

$$C_t = M_t - r_t^{ak} k_t. \tag{17}$$

Substituting (17) into (13) leads to

$$r_t^{ak} = \psi(M_t - r_t^{ak} k_t, \mathbf{p}_t, k_t), \tag{18}$$

which could be used to solve for k_t:

$$k_t = k^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}). \tag{19}$$

Substituting (19) into (17), we obtain

$$C_t = M_t - r_t^{ak} k^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}) = C^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}). \tag{20}$$

This expression together with (19) could be used to derive a system of long-run demand functions for nondurable goods:

$$q_{it} = g_i^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}) = g_i\left[C^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}), \mathbf{p}_t, k^{LR2}(M_t, \mathbf{p}_t, r_t^{ak})\right], \ i = 1, ..., n. \tag{21}$$

Once the long run demands for durable and non-durable goods are defined [see (19) and (21)], we are able to derive the long-run elasticity equations for durable and nondurable goods conditional on total expenditure.

The long-run demand functions in (19) and (21) can be derived from an indirect utility function $\nu(M_t, \mathbf{p}_t, r_t^{ak})$ defined as

$$\nu(M_t, \mathbf{p}_t, r_t^{ak}) \equiv \max_{\mathbf{q}_t, k_t}\left\{u(\mathbf{q}_t, k_t) \;\middle|\; \mathbf{p}_t \cdot \mathbf{q}_t + r_t^{ak} k_t \le M_t\right\}. \tag{22}$$

Application of Roy's identity to (22) gives

$$q_{it} = g_i^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}) = -\frac{\partial \nu(M_t, \mathbf{p}_t, r_t^{ak}) / \partial p_{it}}{\partial \nu(M_t, \mathbf{p}_t, r_t^{ak}) / \partial M_t}, \ i = 1, ..., n,$$

and

$$k_t = k^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}) = -\frac{\partial \nu(M_t, \mathbf{p}_t, r_t^{ak}) / \partial r_t^{ak}}{\partial \nu(M_t, \mathbf{p}_t, r_t^{ak}) / \partial M_t}.$$

The relation between the short-run and long run indirect utility functions defined in (1) and (22) is given by

$$\nu(M_t, \mathbf{p}_t, r_t^{ak}) = \nu\left[C^{LR2}(M_t, \mathbf{p}_t, r_t^{ak}), \mathbf{p}_t, k^{LR2}(M_t, \mathbf{p}_t, r_t^{ak})\right]. \tag{23}$$

Interestingly, Equation (22) suggests an alternative formulation of intertemporal two-stage budgeting. This equation forms the second stage problem of the two-stage budgeting procedure, which describes the intratemporal allocation of *total expenditure* between nondurable and durable goods. By rewriting (4) as $A_s = (1 + r_{s-1})A_{s-1} + Y_s - M_s$, for all $s \ge t$, the first stage problem is solved with respect to M_s, yielding an Euler equation (6b) or (8) expressed in terms of M_s, which describes the intertemporal allocation of total expenditure.

2.5. Risk Aversion

In the presence of uncertainty, the consumer's attitude toward risk, measured by the degree of risk aversion, determines his decisions about occupation, asset allocation, health-related conduct, and moving and job change decisions. The degree of relative risk aversion (*RRA*) is typically measured with the well-known power or CRRA utility function, $u(c_t) = \dfrac{c_t^{1-\zeta} - 1}{1-\zeta}$, where c_t represents the real nondurable consumption, which gives *RRA* = ζ (see Hansen and Singleton, 1983; Mehra and Prescott, 1985). This measure of RRA hinges on restrictive preferences with real consumption under homothetic preferences, and its value is constant. We generalize the measure of risk aversion under nonhomothetic preferences with allowance for relative prices in consumption.

The well-known measures of risk aversion a la Arrow and Pratt are, essentially, a static concept constructed under the assumption that initial wealth is non-random or the consumer has full access to the capital market. Since the consumer cares about consumption, which is directly related to wealth, the indirect utility function (1) could be deployed to construct operational measures of risk aversion (Deschamps, 1973). However, while the demand functions are determined by an ordinary utility function, a risk aversion function is determined by a cardinal utility function. To allow for this, we take a Box-Cox transformation of the long run indirect utility function given in (22):

$$\mathrm{U}(M_t\ \mathbf{p}_t, r_t^{ak}) = \left[\frac{v(M_t\ \mathbf{p}_t, r_t^{ak})^{1-\zeta} - 1}{1-\zeta}\right].$$

The coefficient of relative risk aversion (RRA) is then defined as

$$RRA(M_t\ \mathbf{p}_t, r_t^{ak}) \equiv -\frac{\partial \ln U_M(M_t\ \mathbf{p}_t, r_t^{ak})}{\partial \ln M_t} = -\frac{M_t U_{MM}(M_t\ \mathbf{p}_t, r_t^{ak})}{U_M(M_t\ \mathbf{p}_t, r_t^{ak})}, \tag{13}$$

where $U_M = \dfrac{\partial U(M_t\ \mathbf{p}_t, r_t^{ak})}{\partial M_t}$, $U_{MM} \equiv \dfrac{\partial U_M(M_t\ \mathbf{p}_t, r_t^{ak})}{\partial M_t} = \dfrac{v_{MM}(M_t\ \mathbf{p}_t, r_t^{ak})}{v(M_t\ \mathbf{p}_t, r_t^{ak})^{\zeta}} - \zeta\dfrac{\left[v_M(M_t\ \mathbf{p}_t, r_t^{ak})\right]^2}{v(M_t\ \mathbf{p}_t, r_t^{ak})^{(\zeta+1)}}$,

$v_M = \dfrac{\partial v}{\partial M_t}$ and $v_{MM} = \dfrac{\partial^2 v}{\partial M_t^2}$. The concavity of $\mathrm{U}(M_t\ \mathbf{p}_t, r_t^{ak})$ with respect to M_t implies that $U_{MM} < 0$ and hence $RRA \geq 0$.

2.6. Intertemporal Substitution in Nondurable and Durable Consumption

In the intertemporal optimization problem stated in (5), nondurable consumption and durables stock are choice variables. In practice, it may not be feasible to obtain a structural or closed form solution of these variables from the intertemporal optimization problem, even for simple utility functions when the environment is stochastic. To circumvent this problem, it is a common practice to work with the Euler equation in studies on consumption and saving (see, e.g., Hansen and Singleton, 1983; Ludvigson and Paxson, 2001), which is adopted here. To do so, we first use the Euler equation for nondurable consumption (8) and exploit a lognormal property. Assuming that the quantity (μ_{t+1} / μ_t) has a lognormal distribution and taking logs on both sides of (8), we have

$$\ln\left(\frac{1+r_t}{1+\rho}\right) + E_t(\Delta \ln \mu_{t+1}) + \frac{1}{2}\mathrm{var}_t(\Delta \ln \mu_{t+1}) = 0, \tag{24}$$

where $\Delta \ln \mu_{t+1} = \ln(\mu_{t+1} / \mu_t)$; that is the growth rate of the marginal utility of nondurable consumption. Rearranging this equation gives

$$\Delta \ln \mu_{t+1} = -\ln\big((1+r_t)/(1+\rho)\big) - (1/2)\sigma_{t+1}^2 + e_{t+1}, \tag{25}$$

where $\sigma_{t+1}^2 \equiv \mathrm{Var}_t(\Delta \ln \mu_{t+1})$ capturing the effect of uncertainty in nondurable consumption, and e_{t+1} is an expectation error at time *t*+1 that is uncorrelated with variables known at time *t*.

To evaluate (25), we need an expression for $\Delta \ln \mu_{t+1}$. Logarithmically totally differentiating the marginal utility function of nondurable consumption (6a) (whose arguments are C_t, $\mathbf{p}_t$ and k_t) with respect to time and taking a discrete approximation of log changes, we have

$$\Delta \ln \mu_{t+1} \approx b_{ct}\Delta \ln C_{t+1} + \sum\nolimits_{j=1}^{n} b_{jt}\Delta \ln p_{jt+1} + b_{kt}\Delta \ln k_{t+1}, \tag{26}$$

where

$$b_{ct} \equiv \frac{\partial \ln \mu_t}{\partial \ln C_t} = -\zeta\frac{\partial \ln \nu(C_t, \mathbf{p}_t, k_t)}{\partial \ln C_t} + \frac{\partial \ln \nu_C(C_t, \mathbf{p}_t, k_t)}{\partial \ln C_t},$$

$$b_{jt} \equiv \frac{\partial \ln \mu_t}{\partial \ln p_{jt}} = -\zeta\frac{\partial \ln \nu(C_t, \mathbf{p}_t, k_t)}{\partial \ln p_{jt}} + \frac{\partial \ln \nu_C(C_t, \mathbf{p}_t, k_t)}{\partial \ln p_{jt}}, \quad j = 1, \ldots, n,$$

and

$$b_{kt} \equiv \frac{\partial \ln \mu_t}{\partial \ln k_t} = -\zeta\frac{\partial \ln \nu(C_t, \mathbf{p}_t, k_t)}{\partial \ln k_t} + \frac{\partial \ln \nu_C(C_t, \mathbf{p}_t, k_t)}{\partial \ln k_t}.$$

Since the marginal utility of nondurable consumption is decreasing in C_t and $\mathbf{p}_t$, we expect that $b_{ct} < 0$ and $b_{jt} < 0$ for $j = 1,...,n$, although the sign of b_{kt} is not known a priori. If nondurable consumption and durable goods are substitutes (or complements) in the sense that increasing use of durable goods reduces (or raises) the marginal utility of nondurable consumption, then $b_{kt} < 0$ (or $b_{kt} > 0$). If nondurable consumption is independent of durable goods, then b_{kt} is equal to zero.

Substituting (26) into (25) and solving for $\Delta \ln C_{t+1}$, we obtain a log-linearized Euler equation for nondurable consumption growth:

$$\Delta \ln C_{t+1} = d_{0t}^c + d_{rt}^c \ln(1+r_t) + \sum_{j=1}^n d_{jt}^c \Delta \ln p_{it+1} + d_{kt}^c \Delta \ln k_{t+1} + d_{\sigma t}^c \sigma_{ct+1}^2 + u_{t+1}^c, \tag{27}$$

where $d_{0t}^c \equiv \ln(1+\rho)/b_{ct},\ d_{rt}^c \equiv -1/b_{ct},\ d_{jt}^c \equiv -b_{jt}/b_{ct},\ j = 1,...,n,\ d_{kt}^c \equiv -b_{kt}/b_{ct},\ d_{\sigma t}^c \equiv -b_{ct}/2$ and $u_{t+1}^c \equiv e_{t+1}/b_{ct}$. Note that ln(1+$r_t$) in (27) can be approximated by r_t, i.e., ln(1+r_t) ≈ r_t. In (27), durables stock growth $\Delta \ln k_{t+1}$ is not exogenous but is determined endogenously in the consumer's optimization problem together with $\Delta \ln C_{t+1}$. To allow for the endogeneity of $\Delta \ln k_{t+1}$, we assume that the quantity (μ_{t+1}^k / μ_t^k) in (14) has a lognormal distribution with $\Delta \ln \mu_{t+1}^k = \ln(\mu_{t+1}^k / \mu_t^k)$, the growth rate of the marginal utility of durables stock. Following the above discussion on nondurable consumption growth in (26), we could derive the log-linearized Euler equation for durables stock growth:

$$\Delta \ln k_{t+1} = d_{0t}^k + d_{rt}^k \ln(1+r_t) + \sum_{j=1}^n d_{jt}^k \Delta \ln p_{it+1} + d_{kt}^k \Delta \ln r_{t+1}^{ak} + d_{ct}^k \Delta \ln C_{t+1} + d_{\sigma t}^k \sigma_{kt+1}^2 + u_{t+1}^k, \tag{28}$$

where $\sigma_{kt+1}^2 = \mathrm{Var}_t(\Delta \ln \mu_{t+1}^k)$ capturing the effect of uncertainty in durable consumption.

Equations (27) and (28) constitute a system of two simultaneous equations to solve for $\Delta \ln C_{t+1}$ and $\Delta \ln k_{t+1}$ in terms of exogenous variables. In reduced form, they are given by

$$\Delta \ln C_{t+1} = \phi_{0t}^c + \phi_{rt}^c \ln(1+r_t) + \sum_{j=1}^n \phi_{jt}^c \Delta \ln p_{it+1} + \phi_{kt}^c \Delta \ln r_{t+1}^{ak} + \phi_{\sigma ct}^c \sigma_{ct+1}^2 + \phi_{\sigma kt}^c \sigma_{kt+1}^2 + \omega_{t+1}^c, \tag{29}$$

and

$$\Delta \ln k_{t+1} = \phi_{0t}^k + \phi_{rt}^k \ln(1+r_t) + \sum_{j=1}^n \phi_{jt}^k \Delta \ln p_{it+1} + \phi_{kt}^k \Delta \ln r_{t+1}^{ak} + \phi_{\sigma ct}^k \sigma_{ct+1}^2 + \phi_{\sigma kt}^k \sigma_{kt+1}^2 + \omega_{t+1}^k. \tag{30}$$

These equations identify the relevant variables determining the intertemporal allocations of nondurable and durable consumption including the time preference, interest rate, relative prices of nondurable goods, user cost of durable goods, and the conditional variances capturing uncertainty of nondurable and durable consumption. The coefficients ϕ_{rt}^c and ϕ_{rt}^k are the elasticities of intertemporal substitution for nondurable consumption $\left(EIS_t^C\right)$ and durable goods $\left(EIS_t^k\right)$ respectively. In particular, they are defined as

$$EIS_t^C \equiv \partial \Delta \ln C_{t+1} / \partial \ln(1+r_t) = \phi_{rt}^c$$

and

$$EIS_t^k \equiv \partial \Delta \ln k_{t+1} / \partial \ln(1+r_t) = \phi_{rt}^k.$$

3. Summary and Conclusion

Durable goods are, by and large, ignored in traditional studies in consumer demand and aggregate consumption. Studies that incorporate them often treated them as irreversible or costlessly reversible with no adjustment costs. However, there is an important role of a secondary market in mitigating irreversibility of durable goods with partial or costly reversibility, and adjustment costs are important in consumer behavior. In this paper, we have studied the user cost of durable goods and provided a unified treatment of costly reversibility and adjustment costs with disaggregate nondurable consumption.

There are limited studies that utilize the integrated framework of durable goods in consumer demand and consumption. Kim and Wong (2022a) estimate the model to analyze the demands for durable and nondurable goods by evaluating the effect of adjustment costs, using U.S. data. They find strong evidence for adjustment costs of durable goods in consumer behavior, and the failure to account for them results in a biased inference. In particular, the consumer's observed behavior of durable goods, though not optimal, does not depart substantially from the time path of durables stock estimated with adjustment costs. Income elasticities for nondurable and durable goods estimated with the user cost without adjustment costs reveal a marked difference from those obtained with adjustment costs.

There is a large body of empirical studies undertaken that estimate the elasticity of intertemporal substitution (see Havranek, 2015 and Thimme, 2017, for a survey of evidence). These studies are typically conducted using data on nondurable consumption, but there is a limited number of studies on intertemporal substitution using data on durable and nondurable consumption (see, e.g., Mankiw, 1985; Ogaki and Reinhart, 1998; Yogo, 2006; Pakoš, 2011). The evidence is mixed for the degree of intertemporal substitution in consumption. Kim and Wong (2022b) provide a more complete analysis of intertemporal substitution in nondurable consumption and durable goods with some new evidence.

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