Chapter 3
The Relationships Between Task Design, Anticipated Pedagogies, and Student Learning

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3.1 Introduction

This chapter seeks to synthesize research and scholarship about the relationship between the design of classroom tasks, the pedagogies associated with the effective implementation of tasks, and the learning of mathematics. We use the term *classroom tasks* similarly to Watson and Sullivan (2008) who describe tasks as the questions, situations, and instructions that might be used when teaching students. Tasks prompt activity which offers students opportunities to encounter mathematical concepts, ideas, and strategies. The role of the teacher is to select, modify, design, redesign, sequence, implement, and evaluate the tasks.

The intended task and the enacted task may differ considerably. Even though, as argued by Hiebert and Wearne (1997), "what students learn is largely defined by the tasks they are given" (p. 395), Christiansen and Walther (1986) note that "even when students work on assigned tasks supported by carefully established educational contexts and by corresponding teacher-actions, learning as intended does not follow automatically from their activity on the tasks" (p. 262). Christiansen and Walther differentiate between the task as set and the activity that follows, including

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students' interpretations of the purpose of the task, ways of working, teacher interventions, how language and symbols are used, and what are seen as valuable mathematical actions. The relationship between task and activity can develop in a variety of ways: in some cultures it is the norm for teachers to take a given task and develop it into a whole lesson plan with challenging goals, whereas Stein, Grover, and Henningsen (1996) observe that teachers and students can also act together to reduce a classroom task to a mere sequence of actions.

This chapter first addresses factors that influence the task design process and accompanying pedagogical considerations. It then presents three tasks chosen to provide context for various discussions within the chapter, along with a consideration of the age range of students at which tasks are appropriate. Subsequent sections describe:

1. The *interactions among aspects of task design*: design elements of tasks, the nature of the mathematics that is the focus of the tasks, and the task design processes
2. *Pedagogies*: the nature of the authority and autonomy of the teacher in creating and implementing tasks and problematic aspects of converting tasks from one culture to another
3. *Student learning*: consideration of students' responses in anticipating the pedagogies

A crosscutting theme is that tensions occur in making decisions on culture, mathematics, language, context, and pedagogy. Designers and teachers make decisions among competing options at both the design and implementation stages. In many cases, the decisions on whether, for example, to foster challenge or success, to focus on abstract mathematical ideas or their applications, on whether to exemplify the dominant culture or to introduce perspectives of marginalized groups, may be secondary to the teacher's awareness of those decisions and his/her capacity to interact with the students to explore all aspects of the task potential.

3.2 Factors Influencing Task Design and Pedagogies

Of course, tasks do not exist separately from the pedagogies associated with their use nor are the pedagogies independent of the task. Knowledge for teaching mathematics and anticipatory pedagogical decision-making are two key and complementary elements that are central issues in task design. Jaworski (2014), for example, in elaborating an inquiry stance by teachers in her projects, described a difference between didactics and pedagogy as often used in Europe. She described didactics as being about "the transformation of the subject (mathematics) into activity and tasks through which learners can gain access to the mathematics, engage with mathematics, and come to know mathematical concepts" (p. 2). She described pedagogy as about "creating the learning environment through which learners' engagement with mathematics can take place effectively" (p. 2). Clearly the process of connecting task design with pedagogy involves consideration of both aspects. These are elaborated further in the following sections.

3.2.1 Knowledge for Teaching Mathematics that Informs Task Design

In describing teacher knowledge, we present the categorization proposed by Hill, Ball, and Schilling (2008) who described two aspects of knowledge associated with converting tasks for the use in one's classroom: subject matter knowledge and pedagogical content knowledge. Included within the former are common content knowledge, specialized content knowledge, and knowledge at the mathematical horizon. Included within the latter are knowledge of content and teaching, knowledge of content and students, and knowledge of curriculum.

Perhaps the most critical for task design is *specialized content knowledge* or "the knowledge that allows teachers to engage in particularly *teaching* tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" (Hill et al., 2008, p. 378). Also important is what Hill et al. (2008) described as *knowledge of content and teaching*, including an understanding of how to sequence particular content for instruction, and how to evaluate instructional advantages and disadvantages of particular representations and of the knowledge required to make "instructional decisions about which student contributions to pursue and which to ignore or save for a later time" (p. 401).

These perspectives on teacher knowledge also inform decisions on the placement and contribution of tasks to sequences of learning. Decisions on sequences of learning can be informed by what Simon (1995) described as a *hypothetical learning trajectory* (see also Chap. 2) that:

> provides the teacher with a rationale for choosing a particular instructional design; thus, I (as a teacher) make my design decisions based on my best guess of how learning might proceed. This can be seen in the thinking and planning that preceded my instructional interventions … as well as the spontaneous decisions that I make in response to students' thinking. (pp. 135–136)

Simon noted that such a trajectory is made up of three components: the learning goal; the activities to be undertaken; and a hypothetical cognitive process, "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136). These predictions are not related to sequences of explanations but for students to engage in a succession of problem-like tasks, based on recognition that learning is a product of activity that is "individual and personal, and … based on previously constructed knowledge" (Ernest, 1994, p. 2).

In planning and teaching, the role of the teacher is to identify potential and perceived blockages, prompts, supports, challenges, and pathways. In other words, learning occurs as a product of students working on tasks purposefully selected or designed by the teacher and contributing to ongoing interaction with the teacher and their peers on their strategies and products.

3.2.2 Anticipatory Pedagogical Decision-Making

Based on their knowledge of mathematics and pedagogy, teachers make decisions in anticipation of how students will respond to tasks. Gueudet and Trouche (2011), for example, in elaborating the complex factors informing task implementation, noted the potential gap between the availability of resources, in this case the tasks, and the ways that teachers anticipate the tasks for use in classrooms, which we consider as the pedagogy. They described *documentational genesis* as the two-way processes by which tasks are not only interpreted by teachers but also influence the decisions that teachers make (see also Chap. 6). Gueudet and Trouche (2011) described the use of task as a combination of the task as designed and a *scheme of utilization* which "integrates practice (how to use selected resources for teaching a given subject) and knowledge on mathematics, on mathematics teaching, on students, and on technology" (p. 401), i.e., integrating didactics and pedagogy.

We interpret *scheme of utilization* to be similar to the task elements described by Sullivan et al. (2014) who suggested that, when designers communicate with teachers about the intentions and potential of tasks, this can include indications of the mathematical purpose, ways that tasks can be differentiated, and suggestions of actions that can follow the task to implement the learning. In particular, Sullivan and colleagues proposed a scheme of utilization for the type of task and lesson they were describing to include: one or more challenging task(s); one or more additional task(s) that help to consolidate the learning from the earlier ones; preliminary experiences that are prerequisite but which do not detract from the challenge of the tasks; and supplementary tasks that offer the potential for differentiating the experience through the use of enabling prompts and extending prompts (see Sullivan, Mousley, & Jorgensen, 2009). The term *scheme of utilization* emphasizes that advice on anticipated pedagogical actions is not intended as a script but as a prompt to teachers' own decision-making. Another example of a pedagogical scheme is the *five practices* of Smith and Stein (2011) for orchestrating productive mathematics discussions: anticipating, monitoring, selecting, sequencing, and connecting. These five practices are useful in providing a framework for facilitating rich discussions that mathematics teachers may want to see in their classrooms. These schemes suggest a somewhat overlapping boundary between the design and implementation of tasks, lessons, and sequences.

3.3 Some Tasks Presented to Inform Subsequent Discussions

Three tasks are presented in this section to help exemplify and support the discussions throughout the chapter. The first task is an example of the type of task commonly used as part of the Japanese Lesson Study process (see Fernandez & Yoshida, 2004; also Chaps. 2 and 9 in this volume). The second task was described by Bartolini Bussi, Sun, and Ramploud (2013) who reported on its use, initially developed in a Chinese textbook, in Italy. The third task was described by Peled (2008) and subsequently adapted for use as part of a task implementation project (see Sawatzki & Sullivan, 2015).

There are other types of tasks that could have been chosen, such as mathematical investigations intended to be undertaken independently from the teacher, games that illustrate particular mathematical concepts, and matching of different representations of concepts. The particular three tasks we have chosen are intended to be neither exemplary nor representative, but are provided to allow illustration of issues raised in discussions among contributors to this chapter.

3.3.1 The L-Shaped Area: A Lesson from Japan

This *L-Shaped Area* lesson is representative of tasks used as the basis of lessons in Japanese Lesson Study. The intent is to introduce the notion that the number of squares in a rectangular array can be calculated by multiplying the number of rows by the number of columns.

It can be assumed that teachers might establish a context for the area concepts, such as tatami mats, which are traditional rice-straw mats, 90 cm by 180 cm, used commonly as floor covering and sometimes used to describe floor size (area) of rooms or large buildings. One approach is to present students with a worksheet on which there are two copies of the diagram in Fig. 3.1.

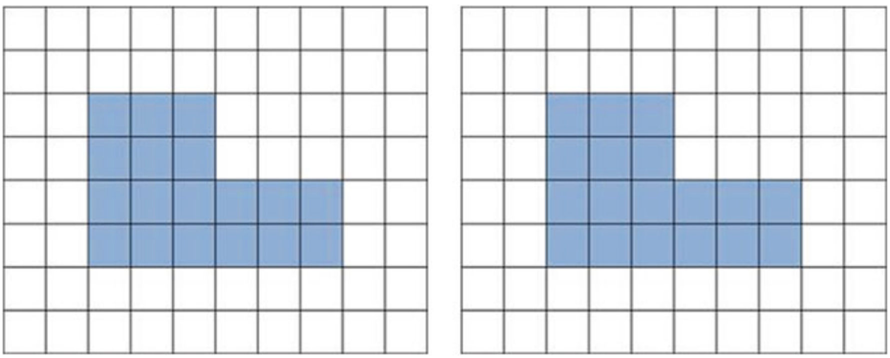


Fig. 3.1 The diagrams used in the L-Shaped Area lesson

The two copies of the diagram on the worksheet communicate to students that, even if they find one solution by counting, a further strategy is expected. Some of the possible solutions include that there are four different ways in which the shape can be rearranged to form a rectangle and the possibility of forming the encompassing rectangle and subtracting the unused portion.

A key phase of such a lesson is the orchestration of selected students' reporting on their strategies, noting that the teacher would have anticipated the types of solutions that students might offer. This would be part of the scheme of utilization of such a task and connects directly to the Smith and Stein (2011) five practices.

There are some interesting characteristics of this lesson: the obvious focus on student-generated strategies; the purposeful choice of students to present and explain their solutions, ensuring that a range of strategies are presented and discussed; and the affordance of a diversity of strategies and representations, allowing students to experience important mathematical ideas.

3.3.2 A Set of Worded Questions

The *Worded Questions* task was described by Bartolini Bussi et al. (2013) who reported on a cross-cultural collaborative project exploring task design and use in both China and Italy (described in more detail below). The set of questions developed in a Chinese textbook are presented in translated form in Fig. 3.2.

The main task is the requested explanation at the top of Fig. 3.2 which may be deduced from consideration of the nine accompanying situations. Note the similarities and differences in the form of the sets of questions and that the numbers are beyond the usual arithmetic range for children in the first years of school in some textbooks. The authors used the task with children aged 8–9, but the task could be presented to older children even in the current format. The framework of questions was similar to that developed as part of the Cognitively Guided Instruction (CGI) project (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). The CGI framework was developed after a project involving researchers and teachers and adhered to two main tenets: that instruction should focus on problem-solving and that teachers should encourage the use of multiple strategies, listen to student reasoning, and build on what students know. What is different, however, is that, in this case, the collection of problems is given simultaneously and not sequentially. Thus, completion of the task requires a holistic or whole-problem view and attention to similarities and differences among the individual tasks.

The set of questions was described by Bartolini Bussi et al. (2013) as follows:

> This is a system of nine problems concerning addition and subtraction, where the organization in rows refers to … combine, change, compare categorization and the organization in columns refers to the same arithmetic operation (either addition or subtraction, …). In each row there is a problem (in the shaded cell) and two variations.
>
> The task is very complex and requires the students not only to solve each problem but also to explain why the nine problems have been arranged in this way. Each problem is associated with a graphic scheme that models on one or two lines the relationship between quantities. (p. 553)

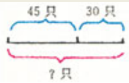
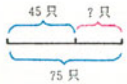
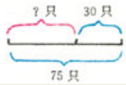
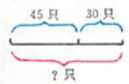
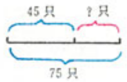
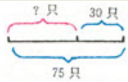
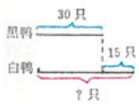
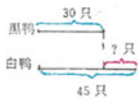
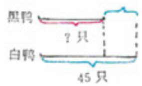
First solve the nine problems below. Then explain why they have been arranged in rows and columns in this way, finding relationships.		
(1) In the river there are 45 white ducks and 30 black ducks. All together how many ducks are there?	(2) In the river there are white ducks and black ducks. All together there are 75 ducks. 45 are white ducks. How many black ducks are there?	(3) In the river there are white ducks and black ducks. All together there are 75 ducks. 30 are black ducks. How many white ducks are there?
		
(1) In the river there is a group of ducks. 30 ducks swim away. 45 ducks are still there. How many ducks are in the group (at the beginning)?	(2) In the river there are 75 ducks. Some ducks swim away. There are still 45 ducks. How many ducks have swum away?	(3) In the river there are 75 ducks. 30 ducks swim away. How many ducks are still there?
		
(1) In the river there are 30 black ducks. White ducks are 15 more than black ducks (black ducks are 15 less than white ducks). How many white ducks are there?	(2) In the river there are 30 black ducks and 45 white ducks. How many white ducks more than black ducks (How many black ducks less than white ducks)?	(3) In the river there are 45 white ducks. Black ducks are 15 less than white ducks (white ducks are 15 more than black ducks). How many black ducks are there?
		

Fig. 3.2 The nine worded questions (Bartolini Bussi, Canalini & Ferri, 2011)

The intention is that the small variations across each row, and also the variations along the columns, allow students to focus on the key elements of the initial problem and the ways that the small variations change the problem and its representation. The nine worded questions construct a three-by-three table and appear as one problem.

From the perspective of variation theory (defined in Chap. 2 and elaborated in Chap. 5), the context of the nine worded questions remains invariant, namely, two groups of ducks in a river, and so is the representation of the part-part-whole relation. In the application of variation theory to task design, mathematics tasks should be designed so that the key idea is varied, allowing learners to experience the effect of the variation in the examples. The known and unknown quantities are varied in each question. The learner's awareness is directed to the pattern of the questions issued in the table. The mathematical aspects, including establishing the relationship between addition and subtraction and also differentiating between forms of subtraction (such as take away and difference), is the focus of the task. Reflection on the individual variations is a more important aspect of this task than the outcomes. Decisions about the use of such a task are a product of teacher knowledge, the scheme of utilization, and the anticipated, hypothetical, learning trajectory.

3.3.3 Shopping for Shoes

The following task based on two related questions is adapted from Peled (2008) and is referred to in the following text as the *Shopping task*. The first question is posed as follows:

> Jenny and Carly go shopping for shoes. Jenny chooses one pair for $110 and another for $100. Carly chooses a pair that costs $160. When they go to pay, the assistant says that there is a sale on and they get 3 pairs of shoes for the price of 2 pairs (the free one is the cheapest).
>
> Give two options for how much Jenny and Carly should each pay. Explain which of these options is fairer.

The second question is posed similarly except that Carly's shoes cost $60 in that question.

Although this task (meaning the two questions together) might seem at first glance to contain little mathematics, the range of arguable solutions is interesting. For example, students have responded to the first question by suggesting that Carly pay $90 (one third of the total revised overall price), or $110 (sharing the $100 overall savings equally), or $126.67 (reducing the price by one third of the $100 overall savings), or $135 (sharing the overall revised cost equally). The task can be used with upper primary students but is also suitable for junior secondary students if solutions based on proportional reasoning are prompted ($116.76—Carly's fraction of the total value of the 3 pairs of shoes multiplied by the actual total cost). The task for students is not so much to determine one possible answer, but to find a way to resolve the differences between these alternatives. Further, not all of these strategies are applicable to the second task. Some of those solution strategies result in Carly being asked to pay more than her shoes cost.

The tasks raise issues about what constitutes "fairness" (note the parallel with the origins of probability theory) and the ways that social considerations, such as friendship, are integral aspects of the solution. In addition, the task makes it clear to students that they can explain and/or defend a particular answer. It emphasizes argumentation and justification, and because of the degree of ambiguity, it allows consideration of social/cultural mediation in mathematics. Of course, the context is culturally laden, and teachers may choose to adapt the task to a situation and cost familiar to and relevant for the students. Again, the use of the task is dependent on the way that the teacher interprets the mathematical potential and the ways the teachers interpret the intended scheme of utilization.

3.3.4 Determining the Age Range of Students for Which These Tasks Are Appropriately Posed

The potential and appropriateness of these three tasks depend on the prior experiences of the students, pedagogic purpose, and teacher and student expectations. The age range at which these three tasks are best suited also depends on the number combinations used and whether the tasks are posed near the start or end of a relevant sequence of tasks. The L-Shaped Area task has been used with students around age 9, the Worded Questions was used with students of ages 8–9, and the Shopping task is intended for students around age 11 but could be used with older students to motivate the idea of proportional reasoning.

In all cases the tasks are somewhat generic and can be adapted for different levels by minor adjustments of particular task aspects. The appropriateness of the age ranges at which the tasks are posed is also a feature of the expectations for the students. Jaworski, Goodchild, Eriksen, and Daland (2011), for example, describe a task that was simply posed but readily adapted for the use by students anywhere between year 1 and 12. In other words, the grade level for tasks is dependent on a range of contextual factors and is not inherent in the task as designed.

3.4 Design Elements of Tasks

One of the recurring themes in the discussions at the ICMI Study Conference that generated this book was that, while the mathematics exemplified by the task is central, there are many other important considerations in designing tasks, especially when designers wish to anticipate and encourage particular pedagogical choices. Our discussion on task design elements is presented in two parts: first, five task design dilemmas are presented to indicate the range of design considerations; and second, we present six suitability criteria that are intended to facilitate analysis of tasks as well as research on and evaluation of those tasks.

3.4.1 Five Dilemmas

Recognition of inherent tensions is central to the decisions that arise in task design and the associated pedagogies. In delineating decisions on elements of tasks, Barbosa and de Oliveira (2013) focused on various dilemmas associated with designing tasks for groups of learners. They used the dilemmas not only as design considerations but also as ways of evaluating the adequacy of tasks that were designed by teachers in the research project on which they were reporting. There were five dilemmas (or conflicts) identified. In the *Australian Concise Oxford Dictionary*, a dilemma is described as a "situation in which a choice has to be made between two … alternatives". These alternatives represent the extremes of the tensions faced by task designers.

3.4.1.1 Context as a Dilemma

The first dilemma to which Barbosa and de Oliveira (2013) refer arises in the mathematical *context* of tasks, which they describe as ranging from pure mathematics to semi-reality to reality. This dilemma (or more accurately continuum in this case) is, on the one hand, the extent to which tasks are set in a realistic context to maximize engagement of students and, on the other hand, whether the context detracts from the potential of the task to achieve the intended learning. In each of the three tasks previously described, the contexts—the mats representing area as covering, the combinations of ducks on the pond, and the shopping discounts—do not necessarily detract from the mathematics to be learned and in fact help make the potential generalizability of the solutions more accessible. The shoes context has the additional element of raising the social decision-making that is required as well as the discussions about what constitutes "fairness".

It is relevant to note that the use of contexts is far from unambiguous. For example, in a review of the testing system in the United Kingdom, Cooper and Dunne (1998) found that contextualizing mathematics items created particular difficulties for low socioeconomic status (SES) students, so much so that they performed significantly poorer than their middle-class peers, while performance on decontextualized tasks was equivalent. Likewise, Lubienski (2000), in studying the implementation of a curriculum program based on open-ended contextualized problems, found that pupils who preferred the contextualized trial materials and considered them easier all had high SES backgrounds, while most pupils who preferred closed, context-free tasks had low SES. This is a complex issue, and it is not clear whether diminished performance was due to contextualization per se or due to other factors like the particular contexts being unfamiliar and alienating for students in low SES communities, or difficulties in separating contextual knowledge from intended "pure" mathematical actions. In other words, the incorporation of contexts does not necessarily ensure tasks are accessible to all students.

Extending the dilemma on the context of tasks, there is a difference between contexts which can be easily seen to be peripheral and those which are central to the mathematics. For example, the L-Shaped Area task and Worded Questions task can easily be transformed into pure mathematical tasks, or different contexts can be used. In contrast, the context of the Shopping task cannot be minimized, or the task will lose meaning. It is also possible for the context to limit the potential of students to generalize solutions.

3.4.1.2 Language as a Dilemma

The second dilemma is about the *language* of the task and the intended solution. On one hand, mathematical precision is part of the desired learning; on the other hand, clarity for the students is needed to support the learning. The language demand of the L-Shaped Area task is mainly connected to the representation of the potential solutions and so is mathematical. For the Worded Questions, the subtle variations in language exemplify the distinctions between the forms of the question. The language used in the Shopping task may not be clear, and so the task may even need to be modeled or role-played by the teacher, and mathematical and social language is required to explain the "fair" solution. Of course, what constitutes fairness can be context dependent. In each case, it is not the language of the task itself, but the way the language is used and interpreted.

3.4.1.3 Structure as a Dilemma

Barbosa and de Oliveira (2013) described a third dilemma as *structure*, which refers to the degree of openness in tasks. This can be considered as much a function of the task outcome as it is the structure. In this dilemma, the consideration is that specific questions can be posed which, on one hand, scaffold student engagement with a task in a more prescribed way and, on the other hand, allow students greater opportunity to make strategic decisions on pathways and destinations for themselves. Barbosa and de Oliveira (2013) describe this continuum as ranging from more closed to more open. Of course, what constitutes openness is the subject of some debate. For example, Hashimoto and Becker (1999) described three categories of problems: those that use a variety of approaches (that have been described as open-middled—see also Wiliam, 1998); those in which the formulation is open (described as open-started, which is close to problem posing); and those that have a range of solutions (open-ended). The L-Shaped Area task is open-middled in that the focus is on student-devised strategies, and the Shopping task is open-ended in that there is a range of feasible solutions. Although the individual Worded Questions are closed, with just one correct answer, there is openness in the choice of representation and also in the identification of commonalities and differences across the questions in the rows and columns. When presented as a set of problems, the focus for the students is not only in finding the respective answers, but also in identifying, understanding, explaining, and justifying the commonalities and differences.

3.4.1.4 Distribution as a Dilemma

The fourth dilemma, described as *distribution*, refers to selecting content to be focused on in the tasks. This is a function of the cognitive demand of the tasks, described by Smith and Stein (2011) as a hierarchy of classroom tasks that develop from *memorization* to *procedures without connections* to *procedures with connections* to *doing mathematics* tasks. Using this nomenclature, in the L-Shaped Area task, the individual students would be *doing mathematics* when creating their solutions and in considering the solutions of others. When students were answering the individual Worded Questions, they would be performing *procedures with connections*, and when identifying commonalities and differences between the questions, they would be *doing mathematics*. It would be possible to respond to the Shopping task at the level of *procedures without connections*; the extent to which the students engaged in *procedures with connections* or *doing mathematics* would depend on the actions of the teacher.

3.4.1.5 Levels of Interactions as a Dilemma

The fifth dilemma refers to the *levels of interactions* of the participants, meaning between the teachers and the students. This can be interpreted to mean that the task does not exist by itself, but its implementation is influenced by the nature of the intended or anticipated interactions between the teacher and students when they are engaged with the task. This is partly connected to the hypothetical learning trajectory (Simon, 1995) that the teacher has anticipated.

In working on the L-Shaped Area task, the expectation is that students engage with the task first and then discuss the various solutions in small groups, with the teacher and as a class. Similarly, for the Worded Questions, the students would work on the questions, in both the Chinese and Italian contexts, with the teacher leading a critical review of the similarities and differences between and within the rows and the columns. In the Shopping task, the students would formulate their own responses with the essential aspect being the discussion and defense of the various viable solutions.

Designers and teachers confront each of these five dilemmas and make appropriate choices, for each and every task, and teachers may take decisions that were not intended or anticipated by the designer.

3.4.2 Task Suitability Criteria

The dilemmas of task design provide a framework that can be used for analysis of suitability of tasks. Giménez, Font, and Vanegas (2013) provide a suitable framework for analysis of tasks generally.

Giménez et al. (2013) describe *epistemic* suitability as "the extent to which the mathematics taught is 'good mathematics'" (p. 581). Decisions on the mathematics are based on both the local and institutional curriculum and prior experiences of the students. Each of the three tasks previously described addresses important mathematical concepts, although the specific concepts are to some extent dependent on the level at which the tasks are used. This connects directly to the mathematics content knowledge of the teacher, who needs to perceive what mathematics is possible.

Giménez et al. (2013) explain that *cognitive* suitability "reflects the degree to which the teaching objectives and what is actually taught are consistent with the students' developmental potential, as well as the closeness of the match between what is eventually learnt and the original targets" (p. 581). Of course, in our illustrative tasks, the cognitive suitability is mainly a function of the sequencing of these tasks among others and cannot be accurately prescribed out of context, without knowing the objectives of the lesson(s) and what was taught previously. Yet each of the tasks offers a variety of starting points for students. The L-Shaped Area task can be solved by counting methods as well as by more mathematically sophisticated approaches. Students might work on various representations of just one of the Worded Questions, while others might engage in the tasks of comparing and contrasting the questions. The Shopping task is mathematically simple at the level at which it is appropriate, yet contrasting various solutions and arguing which is fair is sophisticated. In other words, all three tasks are adaptable to a level of cognitive demand for which the teacher decides she/he can support the engagement of learners.

Interactional suitability "relates to the extent to which the forms of interaction enable students to identify and resolve conflicts of meaning, and promote independent learning" (p. 581). This is similar to what Barbosa and de Oliveira (2013) described as levels of interaction and can refer to interactions between teacher and students, between the students, and for the student and the task. For the L-Shaped Area task, the nature of the interactions depends on discussions facilitated by the teacher that, for example, compare the solutions. For the Worded Questions, the interactions occur when the teacher encourages the students to contrast the various question forms. The interactional suitability depends on teachers' anticipation of what students could become aware of. For students around age 8 years, the interaction may stop at finding the relationship of addition and subtraction among three quantities (two of them are known and one is unknown). For older students, the interaction may be directed at distinguishing the two different patterns for the subtraction operation. In the second and third columns, one pattern of subtraction is to figure out a partial quantity when the sum and the other partial quantity are known. But the other pattern of subtraction is to compare the difference between a bigger quantity and a smaller quantity. For the Shopping task, the interactions depend on the extent to which the teacher allows and facilitates the consideration of alternatives by the students and prompts discussions about fairness. In other words, each of the tasks has its own scheme of utilization.

Mediational suitability refers to the "availability and adequacy of the material and temporal resources required by the teaching/learning process" (p. 581). The key feature of the L-Shaped Area task is the presentation to students of a worksheet that requires two methods of solution. This is intended to prompt students to offer two different solutions, especially when this has become a normal expectation of both the teacher and the students. The cognitive demand of this task is evidenced by the level and type of engagement. The juxtaposition of the various Worded Questions prompts the students to engage with the questions both one by one (in the Italian implementation) and overall (as in the Chinese model). The context of the Shopping task may require role playing the situation so the nature of the task (not the solution) becomes clear.

Affective suitability reflects the students' degree of involvement (interest, motivation, etc.) in the task. As Middleton (1995) argued, a key task characteristic that influences affective responses is the degree of control, which is better described as the opportunity for student decision-making, meaning the choices that the students can make. Middleton (1995) also suggested that interest and arousal are important determinants of student motivation. In the L-Shaped Area task, the choice is the mode of solution, while the interest in the task is motivated by the use of a familiar context. Although there is limited choice in the individual Worded Questions, students make decisions on ways they interpret the similarities and differences between the questions; in the Shopping task, the choices are the decisions on what is "fair" and imagining themselves in a related situation.

Giménez et al. (2013) considered *ecological* suitability as "the degree of compatibility between the study process and the school's educational policies, the curricular guidelines and the characteristics of the social context, etc." (p. 581). The L-Shaped Area task and the Worded Questions attempt to address multiple solutions to a problem at different cognitive levels often explicit in the mathematics curriculum. The link between the Shopping task and a conventional mathematics curriculum is, however, more tenuous and requires intervention by the teacher for this to become explicit. Often, the aims of mathematics curricula are difficult to discern. In Chap. 5, the complexity of such curricula is described in more detail, with an attempt to categorize the different purposes afforded by text-based tasks.

Overall, this section summarizes two frameworks that describe the elements of, and design considerations for, tasks. They illustrate not only that task design is multidimensional, but also that there are tensions to be considered at each phase of design. The tensions are present for task designers and teacher adaptation, whether they are designing tasks for themselves or for others. The next section focuses on consideration of the mathematical content of tasks.

3.5 The Nature of the Mathematics that is the Focus of the Tasks

Perhaps the most critical element of task design is the potential for the task to prompt the learning of the intended mathematical concepts. But there are different perspectives of mathematics that can be considered. On one hand, Ernest (2010) described the goals of a *practical* perspective of mathematics as students learning the mathematics adequate for general employment and functioning in society, drawing on the mathematics used by various professional and industry groups. Ernest included in this perspective the types of calculations one does as part of everyday living, including best-buy comparisons, time management, budgeting, planning home maintenance projects, choosing routes to travel, interpreting data in the newspapers, and so on.

On the other hand, Ernest described a *specialized* perspective as that mathematical understanding which forms the basis of university studies in science, technology, and engineering. He argued that this includes an ability to pose and solve problems, appreciate the contributions of mathematics to culture, the nature of reasoning, and intuitive appreciation of mathematical ideas such as "pattern, symmetry, structure, proof, paradox, recursion, randomness, chaos, and infinity" (Ernest, 2010, p. 24).

Both perspectives are directly connected to the teachers' mathematical knowledge for teaching and clearly inform task design. In taking a specialized perspective, the following subsection elaborates considerations for tasks that prioritize explicit mathematical goals. In taking a practical perspective, the subsequent subsection explores issues associated with tasks that focus on mathematical literacy, described here as numeracy. As with other design elements, these two can be in tension, in that a focus on one can detract from the goals associated with the other.

3.5.1 Tasks that Address Specialized Mathematical Goals

Connected to the teacher knowledge that informs task design and implementation are two aspects: the conceptual ideas represented by a specialized perspective and the mathematical processes in which the students might be expected to engage.

From this perspective, it is assumed that teachers will be explicit about the nature of the expected mathematical goals for the students, not only as part of their planning but also in their ongoing interactions with students. There is, however, an inherent tension here between articulating a mathematical goal to students and having students discover or investigate a mathematical concept or idea in a lesson. In the latter case, the articulation of the goal needs to be rather general, so as to not reveal the concept to be discovered or investigated. Further, Smith and Stein (2011) argue that articulating the mathematical goals (at the design phase, especially if the design is done other than by the class teacher) can support furthering teacher knowledge of the specialized mathematics.

In the case of the L-Shaped Area task, the mathematical goals include the array model of multiplication, conservation of area, and seeing other ways of calculating area other than counting squares one by one. Experience with the task also lays a foundation for study of later concepts such as area conservation (that is useful in the process of calculating the area of parallelograms), breaking a composite shape into parts (that can inform the calculation of the area of trapezoids), and subtracting areas (that may be used in calculating the area of paths around shapes). The task would be entirely different if the area formula, $A = l \times w$, was made explicit to students as the goal of the task.

In the Worded Questions, the mathematical concepts include the relationship between addition, subtraction, and their representation and the different forms of subtraction (e.g., take away, difference), with generality in recognizing reciprocal relationships between addition and subtraction. Such aspects commonly are emphasized in curriculum statements. In Australia, for example, the content of relevant aspects of the curriculum is presented through statements such as:

> Represent and solve simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts.

The Shopping task does not focus on specific mathematical concepts, unless it is posed in the context of proportionality, in which case the extent to which the proportional allocation of the costs represents fairness in the different tasks can be the focus of discussion.

The specialized perspective, as described by Ernest, also addresses the process goals associated with the tasks. Examples of such process goals for students are:

- Making connections between intuitive knowledge and formal mathematical principles/conventions/ideas
- Developing mathematical modeling and problem-solving skills
- Developing algebraic thinking/the ability to express generality

- Learning that jumping with the first idea that comes to mind is not always a good strategy
- Learning the value of examining multiple solutions to a problem and building connections between those solutions

These process aspects are implied in the L-Shaped Area task by the invitation to students to provide two solution methods. In Worded Questions, the potential for generality is in recognizing reciprocal relationships as well as in the interaction between the question forms. In the Shopping task, the mathematical processes include the justifications of a "fair" solution, comparing solution options, and the applicability of the same solution method to both of the questions.

Such process goals are evident in four strands of mathematical proficiency described by Kilpatrick, Swafford, and Findell (2001). The first strand, *conceptual understanding*, includes the comprehension of mathematical concepts, operations, and relations. The second strand, *procedural fluency*, refers to carrying out procedures flexibly, accurately, efficiently, and appropriately and, in addition to these procedures, having factual knowledge and concepts that come to mind readily. The third strand, *strategic competence*, includes the ability to formulate, represent, and solve mathematical problems. The fourth strand, *adaptive reasoning*, includes the capacity for logical thought, reflection, explanation, and justification.

3.5.2 Designing Tasks that Address a Practical Perspective

Taking a different stance, Goos, Geiger, and Dole (2010) use a model of mathematics focusing on real-life contexts, application of mathematical knowledge, use of representational, physical, and digital tools and that emphasizes cultivation of positive dispositions toward mathematics. Their model, shown in Fig. 3.3, illustrates the considerations associated with tasks.

This model connects various aspects informing task design. Goos, Geiger, and Dole (2013) used the term *mathematical knowledge* to include not only fluency with accessing concepts and skills, but also problem-solving strategies and the ability to make sensible estimations. Such knowledge is accessed in solving the L-Shaped Area task and the Worded Questions. On one level, the mathematical demand of the Shopping task is limited, although it is noted that in junior secondary levels it can be anticipated that some students might propose a solution based on proportionality which would represent that mathematical knowledge.

Goos et al. also proposed *positive dispositions*, "a willingness and confidence to engage with tasks and apply their mathematical knowledge flexibly and adaptively" (p. 591), as part of their model (note that this is also the fifth proficiency from Kilpatrick et al., 2001). One of the elements of disposition is related to the opportunities for students to make decisions on the nature of the solution and the pathway to the solution. This relates to the notions of control and student decision-making. The L-Shaped Area task and the Shopping task both allow such opportunities.

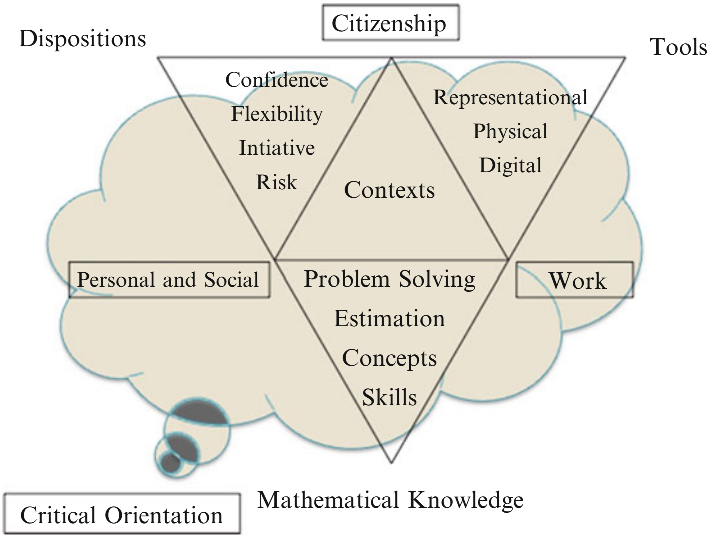


Fig. 3.3 The model proposed by Goos et al. (2010)

Worded Questions has this element when the students are seeking to describe similarities and differences between the questions.

Another element that can foster a positive disposition is if the task has what is described as a "low floor, but high ceiling". Recognizing that it is not clear whether it is the task that fosters the positive disposition or the disposition that facilitates the engagement with the task, all three of the tasks have the potential to foster improved disposition. The "floor" (which in this case refers to the level at which students might initially engage with the task) in the L-Shaped Area task is represented by a solution in which the square units are counted individually, in the individual Worded Questions by making a physical model of the ducks or a number line segment, and in the Shopping task by finding a single possible cost breakdown. The "ceiling" in the L-Shaped Area task is represented by any of the solutions expressed in general form and also perhaps by the articulation of a general solution strategy; in Worded Questions by explaining the similarities and differences between the rows and columns, respectively; and in the Shopping task by contrasting the solutions for the two forms and considering which approaches can apply to both problems and which do not.

A third element presented by Goos et al. (2013) is the tools, which they elaborate as follows:

> In school and workplace contexts, tools may be representational (symbol systems, graphs, maps, diagrams, drawings, tables), physical (models, measuring instruments), and digital (computers, software, calculators, internet). (p. 591)

None of the three tasks require the use of digital technology to assist in formulating solutions, although calculators may be useful for students who might otherwise not be able to engage with the tasks. In the L-Shaped Area task, the two diagrams provide a tool with which to represent the solutions and could be presented as manipulable screen objects. In Worded Questions, one tool could be a number line that is used to represent the actions in solving the task. In the Shopping task, a tool could be the ways that students choose to represent their solutions.

The Goos et al. (2013) model also includes the context, which can range from “personal, citizenship-related, work-related” on one hand to contexts related to the curriculum. In other words, the purpose of posing a particular task might be to explicate ways that the world is mathematized or the purpose might be to address aspects of the intended curriculum. The L-Shaped Area task and Worded Questions are situated at the “curriculum” end of this range, whereas the Shopping task is more at the “personal” end.

A further aspect is a critical orientation to numeracy which Goos et al. describe as appropriate and inappropriate uses of mathematical thinking. A critical orientation is most evident in the Shopping task. Given that arguable solutions range from $90 to $135 (ignoring the solution in which the discount is not shared) in the first situation, the considerations that might inform choices about what the shoppers might pay come to the fore. Indeed, not only explaining the respective solutions but also listening to the explanations of others is connected to developing such a critical orientation. This might include consideration of friendships and the perspective on fairness.

Although it could be argued that tasks designed for supporting the learning of mathematics are more likely to achieve their goal if the mathematics is both important and explicit, this section illustrates that there is tension in resolving the balance between purely mathematical goals and those which are more social or personal or related to illustrating the usefulness of mathematics in making everyday decisions.

3.6 Task Design Processes

Chapter 2 presents an overview of frameworks and principles for task design, analyzing them on different frame levels: grand, intermediate, and domain-specific frames. Clear throughout this discussion is the inextricable relationship between the design of the task and that of the learning environment. These two must be considered simultaneously.

Ron, Zaslavsky, and Zodik (2013) described a three-stage, backward-design process that includes:

- Stating goal(s) and connecting the task to the goal(s)
- Designing a generic task that addresses these goals; and then (when applicable)
- Carefully choosing the specific examples to “plug in” the generic task (p. 641)

One critical aspect of this design process is to provide well-thought-out starting points for teachers. Another aspect is to explore the role of tasks in fostering awareness of the role of tools, in this case meaning the mathematical routines or procedures that can be enlisted in the solution of problems. They argued, "From a design perspective, with respect to tools, we would like teachers to be able to design tasks that foster discussion of the merits and limitation of existing as well as new tools" (p. 642). They continued:

> This constitutes a challenge for teachers and teacher-educators, because on the one hand, we often want to point to the limitation of existing tools for a particular purpose, while we need to maintain the usefulness and merits of the existing tools for other purposes (otherwise students will believe that much of what they learn will need to be abandoned in the future). … Thus, the progression from existing tools/concepts to new tools/concepts should lead to an extended mathematical 'toolbox'. (p. 642)

In their perspective, the focus of the task design was to emphasize to students that they were acquiring new tools for future use, such as maybe learning a formula for area of a rectangle in the L-Shaped Area task or number line use in Worded Questions. An associated perspective was outlined by Chu (2013) who described a process of task design with two components: the task design itself and consideration of a particular context in which students were learning mathematics in a second language (English). The process he outlined started with specific academic and linguistic goals, selection of inputs to tasks, specifying the conditions constraining those inputs, clarifying procedures needed, and predicting outcomes as both products (artifacts produced) and processes (ways of engaging). He argued that:

> This framework shapes activities built around mathematical practices to scaffold student engagement in interactive tasks that foster their emerging autonomy. … Results suggest trajectories for teachers' shifting understanding of conceptual, academic, and linguistic goals as they appropriate a pedagogy of promise that fully develops the potential of all (English language learners). (p. 559)

As Chu explained, one aspect is the design of tasks, and another aspect is the design of the instruction that draws upon and connects those tasks. Chu articulated five principles that guide the design of instructional experiences for students: academic rigor, high challenge/high support, quality interactions, language focus, and well-constructed curriculum.

Knott, Olson, Adams, and Ely (2013) describe a process that focuses on adapting suggestions from texts to inform instruction. They suggested that this process, which they refer to as *turning a lesson upside down*, involves both components—design of the task and consideration of the learning environment:

- Selecting a lesson from a text and identifying the key mathematical understanding or idea
- Writing the mathematical idea as a generalization
- Deciding whether the key understandings entail justification
- Finding or designing a task or sequence of tasks that promotes exploration of the key idea
- Writing questions for students that can prompt them to generalize the key idea

It is important to consider further the process of converting textbook examples to classroom tasks. Particularly in the United States, teachers often take text resources and the associated teachers' guides as the curriculum and plan their teaching from there.

For example, Remillard, Herbel-Eisenmann, and Lloyd (2009) described planning processes as "transforming curriculum ideals, captured in the form of mathematical tasks, lesson plans, and pedagogical recommendations into real classroom events" (p. 1). Stein et al. (1996) described the initial phase of such planning as the teacher taking the mathematical task as presented in instructional materials, before the transformation process.

Peled and Suzan (2013) describe a different process of task design. They focus on what they described as "simple" tasks, meaning tasks that are parsimoniously posed in order to support a shift toward a model of teaching based on problem-solving approaches. They argued that such tasks are preferable to complex modeling tasks because of the resistance of teachers to using such tasks initially. Peled and Suzan (2013) suggested that their tasks serve a double purpose: both to create learning opportunities and to serve as a model for future task creation by teachers. In offering examples of their simple tasks, they wrote:

> One of the tasks involved cutting greeting cards … and the second involved pouring beer from a container into cans. The main and relevant difference between the problems involves the rigidity of the cardboard versus the "flexibility" of liquid. This feature results in different types of "remainders", as the rigid material does not allow remaining scraps to be put together (unlike a situation such as cutting cookies with "flexible" dough). This difference leads to fitting very different mathematical models. (p. 633)

In other words, the process of task design can focus on affective, practical, and/or mathematical aspects, and these foci can be specific or implied. Further, the focus might be on tools and sequences, and this design process influences both the task itself and the learning experiences constructed around the task, including the learning of teachers.

3.6.1 The Role of the Authority and Autonomy of the Teacher in Designing and Implementing Tasks

A further influence on the design and implementation of tasks in classrooms is the role of the teacher either in adapting a task developed by others (as previously indicated) or in designing the task in the first place. Although there is substantial evidence that the implementation of tasks by teachers can subvert the aims of the task's designer, such as by reducing or increasing the demand of the tasks on the students (see, e.g., Desforges & Cockburn, 1987; Prestage & Perks, 2007; Stein et al., 1996; Tzur, 2008), it seems also that involving teachers in consideration of design issues can affect the potential of the tasks. This aspect of task design and implementation is further elaborated on in Chaps. 2, 5, and 6. Here we note two aspects connected directly with task design.

Askew and Canty (2013) collaborated on a task design and teacher learning project. They introduced teachers to broad principles underpinning the tasks and argued that the tasks were:

> …appropriated in ways that may not have matched with the intentions of the designer, but rather than this being an obstacle it led to rich discussions around the nature of teaching

> and learning. … Rather than trying to construct tasks that are 'teacher-proof' or at least supported by materials that explicate in detail the intended style of implementation, that working with the fuzziness of appropriation can be a strength. (p. 531)

Kullberg, Runesson, and Mårtensson (2013, 2014) describe a project in which teacher adaptations of a task enhanced student learning. Rather than seeking to limit teacher adaptations, they fostered and celebrated them. In their learning study approach, Kullberg et al. (2013) focused on the principles of variation (for more on Variation Theory, see Chap. 2) to structure a lesson focused on division by a number between 0 and 1. In their analysis they concluded:

> The case study is an example of a design project where teachers' reflection on their teaching and the learners' responses can lead to a refinement of the task design …, but also to a greater accuracy and clarity about what to point out and make discernible to the learners. (p. 617)

Each of these examples recognizes the central role of the teacher and the teacher's knowledge and learning in the (re)design of tasks and their implementation. Rather than fearing that teacher adaptations may limit the potential of the task, as is assumed by some designers, involving teachers as far as possible in the intentions of the designer can enhance the implementation of the task.

Authorship is considered further in Chap. 5. Here we have not said much about teachers creating tasks for their own use, but in Chap. 4 the notion of emergent task design in response to what takes place in lessons is an important related idea.

3.6.2 Problematic Aspects of Converting Tasks from One Culture to Another

An issue about anticipated pedagogical intentions is the adaptation of a task designed for one culture for the use in a different culture. Culture here is taken in both its broad interpretation as being associated with a different geographic location and language and at times in its narrower interpretation to mean the prevalent cultural context of the subject (mathematics) and of the classroom, including the social and sociomathematical norms in place. This section addresses the former of these, while the latter issues are considered in the subsequent section.

There are a number of key connections between task formulation and culture. These include: the cultural specificity of task context, the relationship between cultural considerations and the types of solution prompted, the precision of the available language and its relationship to mathematics concepts, and the compatibility between the cultural background of the teacher and the students, national traditions, and classroom constraints.

It is not a simple task to take curriculum from one language and culture to use in another. If it were merely a matter of translation rather than transformation, one could seamlessly appropriate curricular materials. But much of the cultural context, especially in mathematics, is implicit, nestled within the sequencing of tasks and activities, the choice of context, and the social and mathematical norms of the specific classroom and of the broader society. Further, both across and within cultures, socioeconomic differences can influence the contexts that are meaningful to students. The culturally and contextually dependent relationships among different mathematical concepts are complex. In short, the successful implementation of specific curriculum and/or tasks depends to a large extent on the teacher as a cultural interpreter. It is the teacher's role to understand and interpret the task as it is contextualized in one culture and re-present it as a culturally relevant and appropriate task in her own context. The mathematical goals of a set of tasks or a piece of curriculum may be quite different across cultures.

To illustrate this point, the L-Shaped Area lesson described earlier used Japanese tatami mats to introduce area as covering; these are not square. In a Western context, the teacher may choose to reinterpret the task to involve carpet or tile squares or some other contextually relevant material. Alternatively, the teacher might use the tatami mats with part of the intention being to include that international dimension in the student experience. The Shopping task, as posed, is specific to higher-income groups, but the notion of "two-for-one" discounts are common, and so it can be expected that teachers adapt contexts to suit their students while preserving the essential elements of the tasks.

In the study that generated the Worded Questions example, Bartolini Bussi et al. (2013) compared and contrasted the cultural contexts of a piece of mathematics curriculum and described the differences in approach, context, sequencing, and understanding between Chinese and Italian teaching cultures. They conducted a study in which Italian teachers reinterpreted the mathematics inherent in the task from the Chinese curricular approach to make it culturally accessible for teaching in Italy. Bartolini Bussi et al. (2013) described how they used a complex task from a Chinese textbook that emphasized the connectedness and complementarity of addition and subtraction and transformed it into several separate tasks, some involving addition and some subtraction. In the Italian curriculum, as in many western societies, the mathematical concepts of addition and subtraction are sometimes taught as separate mathematical concepts, each with its own set of rules. In contrast, in the Chinese tradition those operations are seen as inextricably intertwined representations of the additive relationship, as yin and yang and warp and weft, with understanding developing only by considering the whole fabric. Bartolini Bussi et al. (2013) illustrated a fundamental principle of the Chinese curriculum: "one problem, multiple changes", which emphasizes varying conditions and conclusions. This stands in stark contrast to the western approach of sequencing learning from one concept to a single subsequent concept, with limited emphasis on connections. Bartolini Bussi et al. (2013) described how practicing teachers in their project "re-designed it to tailor it to the Italian tradition and to their individual teaching styles and systems of beliefs" (p. 554). They reported:

> Three main changes were introduced: (1) the single task has been transformed into a set of several tasks; (2) classroom work was organized according to a sequence inspired by the theoretical framework of semiotic mediation after a Vygotskian approach …; (a) individual or small group solution of each row of problems followed by the invention of three problems similar to the given ones, to foster the awareness of the problem structure; (b) collective discussion of the findings, with teacher's orchestration. (p. 554)

In other words, rather than presenting the entire set of questions simultaneously, they were presented sequentially, with the variations emerging progressively. These changes are indicative of the expected ways of teaching in the respective countries. This was also evident in the decisions about representations in the Italian context:

> Moreover the solving graphic schemes (at the beginning) were removed and introduced later, after thorough exploration and solution of the problems, as students were not familiar with such schemes. In this way the use of a graphic scheme was acknowledged by students as meaningful and not perceived as an automatic answer to a given task. (p. 554)

In the Chinese classes, the number line was proposed as a prompt to an alternate representation of a solution. In the Italian classes, the line was removed to avoid it predetermining the solution path chosen by the students.

It is important to note that it is the particular features of the task, deemed important and emphasized by the teacher, that are culturally dependent. In one culture, the emphasis might be on *solving the task and getting an answer*. In another culture, the emphasis might be on the *process or processes by which the task is solved*, and in yet another context, it may be *the connections and patterns that are observed over a set of problems* that are emphasized. The classroom work then would focus on the processes by which different solutions were obtained, and much less emphasis would be placed on the answer itself.

Interestingly, the notion of perspectives on teaching and learning and task design being connected to particular cultures and languages is not restricted to the transfer of tasks across national boundaries. For example, in designing tasks for Australian Indigenous students, not only can the familiarity of the context be considered but also mathematical strengths of the students. Indigenous Australian students have well-developed conceptions of location that can be used in the teaching of more formal geometrical concepts. Further, where the composition of classrooms includes a mix of ethnic, racial, language, and socioeconomic student backgrounds, the differences between the experience and orientation of the respective groups are important design and pedagogical considerations.

3.6.3 Classroom Culture and Anticipated Pedagogies

Student practices and expectations in the classroom depend on the establishment of social and sociomathematical norms. The prevailing classroom culture can have a significant impact on anticipated pedagogies. If, for example, a teacher intends that students replicate routines that have been explicitly demonstrated, then a teacher-directed lesson structure supported by classrooms in which students attend to accuracy and detail is important. If, on the other hand, teachers seek to transfer some responsibility for learning to the students, then different processes and ways of communicating are needed. This is in part a function of the classroom culture and processes that are established over time.

In an important meta-analysis of 49 research studies on classroom culture between 1991 and 2011, Rollard (2012) described three significant and relevant findings that inform the connections between task design and pedagogies. Firstly, the meta-analysis found that the middle years of schooling (ages 9–14 years) are critical for connecting classroom goal structures and the formation of student attitudes because it is in these years that parents and teachers become more interested in assessment of success, and there is more overt competition between students. Students in these years may be more reluctant to engage with tasks that they have not been shown how to do and sometimes avoid the perception of trying hard to avoid censure from other students (see Sullivan, Tobias, & McDonough, 2006). Particularly at these levels, establishing a positive classroom culture is a prerequisite to effective use of some types of tasks.

Secondly, Rollard (2012) concluded that classrooms that promote mastery, meaning those that focus on the learning of the content rather than competitive performance, are more likely to foster positive student attitudes to learning. This is similar to the findings of Dweck (2000) who explained that students who seek mastery of content are more willing to make learning decisions for themselves and are less dependent on the affirmation of others. Such students tend to develop a growth mindset approach to learning, believing that hard work pays off. Rollard (2012) suggested that teachers can actively promote a mastery orientation in the students, in part by paying attention to the type of tasks that are posed and by emphasizing the process rather than the answer in the classroom. Dweck suggests that an emphasis be placed on hard work rather than on intelligence (we describe her ideas more fully below). Thirdly, Rollard concluded from the meta-analysis that classrooms in which teachers actively support the learning of all students promote high achievement and effort.

It is interesting to consider the similarities and differences in Rollard's conclusions and other models of classroom culture. For example, Cobb and McClain (1999) argued that students should have opportunities for "personally experienced mathematical problems … (which) would constitute opportunities for them to learn" (p. 12). They also described the importance of classroom social norms, such as "explaining and justifying solutions, attempting to make sense of explanations given by others … and questioning alternatives when a conflict of interpretations had become apparent" (p. 10). For Rollard and also Cobb and McClain, classroom culture is not created by establishing rules in advance but through the structure of lessons, the types of tasks that are posed, the ongoing interactions between teachers and students during lessons, and the relationship of the students with the teacher.

In another study, Brown and Coles (2013) explored specific ways in which teachers took into account their established classroom cultures when designing tasks, so as to effectively establish or reinforce a desired classroom culture. Teachers considered the student age group, the classroom culture, their prior knowledge and skills, and the place of the task in the larger curriculum. In turn, the design of the task impacted the classroom culture, the students' knowledge and skills, and often the larger curriculum.

Brown and Coles used the term *relentless consistency* to describe the desired teaching orientation needed for supporting student learning. For example, if the desire is to create a classroom environment where children are comfortable struggling with complex, open-ended problems, then time must be spent in establishing this comfort level. Once children are used to this, their expectation that they work in this way persists. Rather than seeking to design tasks that can be implemented as written, teachers make choices that require them to be "relentlessly consistent" about not telling students what to do. This is best achieved by designing and/or using a very familiar task, freeing up the teacher to attend to the consistency of what she values in the students' work.

Of course, classroom culture is also a characteristic of context and the community culture. Examples of relevant factors include the size of the class groups, the flexibility of the furniture, whether the language of instruction is the first language of the students, the classroom resources, and the processes of selecting students for the class. There are also factors related to the overall national cultural context. For example, there are Japanese technical terms that describe the purpose and enactment of various aspects of lessons (see Chap. 2). Such terminology would no doubt assist teachers in establishing classroom ways of working.

Elaborating on this notion of classroom culture, Brown and Coles (2013) described part of the teachers' role as being to create classrooms in which persistence, consideration of alternatives, and justification of reasoning are the norm. Brown and Coles (2013) also argued that establishing such a classroom culture and routines takes time to foster.

> … in designing and implementing tasks, teachers have, as a base for decision-making, the classroom cultures they have already established with their students. These cultures are developed over time from the first lessons with a new group. (p. 623)

Similarly, Chu (2013) addressed pedagogical features of the specialized learning environment for learners of English on which he focused:

> … an architecture of three moments assists teachers in deconstructing broad goals into connected intermediate objectives that flow together smoothly. (p. 561)

These three "moments" are specific phases of a lesson: preparing the learners, interacting with the concept, and extending understanding.

A further perspective on classroom culture is described by GEMAD (2013). Their approach includes an expectation that students will:

> develop their own cognitive strategies, manage different representations of the mathematical concepts, choose the best solution strategies, argue about their decisions and communicate fluently their thinking processes. (p. 570)

GEMAD described the teacher's role in fostering this classroom culture to include making learning goals explicit, prompting groups of students to create their own solutions and to present these solutions to the class, and focusing on "argumentation and justification" (p. 572). An interesting aspect of the classroom culture that the GEMAD project was seeking is their explicit intention that students be grouped heterogeneously.

Effective implementation of the L-Shaped Area task is a product of a classroom culture which had been established earlier using the Japanese Lesson Study process. Although the Worded Questions task might be representative of a conventional approach to teaching, the task of synthesizing across the tasks requires students to take a meta-view of the set of tasks and to explicitly look for and make connections among different but related procedures—a higher level of cognitive demand. This way of working is not established overnight. Similarly, the requirement in the Shopping task for students to consider the options suggested by others and to articulate their own preferences is a product of the classroom norms that have been already established.

If the designer of the task is not the classroom teacher, there is a need to anticipate what the culture of the classroom might bear, at least to suggest what kind of classroom culture and instructional approach might appropriately support the implementation of the designed task or task sequence. Taking the reverse perspective, the teacher can hypothesize the classroom approaches that might best support the implementation of a task. Indeed this emphasizes that tasks cannot be "teacher-proofed" and teachers must make active decisions on the implementation of tasks. In any case, there is always interaction between the task itself and its classroom realization.

3.7 Considering the Students' Responses in Anticipating the Pedagogies

Common to the three illustrative tasks are expectations that students will create mathematical knowledge by engaging with the task with thoughtful support from the teacher. The starting point is generally a task that is appropriately challenging for those who will engage with it and with the potential to be supportive of various mathematical product and process goals and to positively influence affective dimensions of student engagement with the task.

Although many aspects of pedagogy have been addressed in earlier sections, the following seeks to describe some initiating aspects of pedagogy that have not so far been considered. There are three issues: the motivation of the students, the introduction of a task, and differentiating the task to ensure it is accessible to all students.

3.7.1 Student Motivation

The first issue associated and discussed in this section is motivation; goals might include:

- Students enjoy the mathematics they are learning.
- Students see the usefulness of the mathematics to them.
- Students be able to interpret the world mathematically.
- Students see the connection between mathematics learning and their future study and career options.
- Students know that they can learn mathematics if they persist.

Recognizing that all of these are important, and also acknowledging that different readers will have different preferences, it is arguable that the most critical goal is that students come to know that they can learn mathematics. If they do learn, then there is the potential that such learning becomes a lifelong endeavor and not merely a pathway to some possibly unrelated goal. Whether any of these goals are achieved depends on the implementation of the task and the response of the students. Dweck (2000) argues that finding ways to support students is as much connected to their orientation to learning as it is to cognitive approaches. She categorizes students' orientation to learning in terms of whether they hold either *mastery* goals or *performance* goals.

Students with mastery goals, according to Dweck, seek to "master" the content and self-evaluate in terms of whether they feel they can transfer their knowledge to other situations. They remain focused on mastery especially when challenged. Such students do not see failure as a negative reflection on themselves, and they connect effort with success. In contrast, students with performance goals are interested merely in whether their answers are correct. Such students want to learn but are more comfortable on tasks with which they are familiar. They give up quickly when challenged; they evaluate their achievements based on positive feedback from a teacher.

Another motivational factor is the mathematical intention behind the task. For example, the task designer might intend that students will learn particular mathematical concepts, they might apply the mathematics to a social situation, or the goal might be simply to elicit positive motivation of the students by increasing their interest in the result. All three of the illustrative tasks incorporate a mix of such factors. The L-Shaped Area task offers students experience with concepts which have the potential for future use rather than immediate benefits for learning. Similarly, delayed usefulness can claimed for the Worded Questions task. The Shopping task has a potential immediate utility and only if the teacher is able to elicit an effective discussion about processes of determining fairness would the longer-term utility become evident.

3.7.2 Introducing the Task to the Students

A second issue is considering ways of introducing tasks to students. On one hand, teachers want students to be able to interpret the task demands. On the other hand, it is assumed that teachers will not give so much direction to students that it becomes impossible for them to create their own mathematics through working on the task.

Several studies find teachers who somehow reduce the challenge of tasks. Stein et al. (1996), in a classroom-based study of task implementation, noted a tendency for teachers to reduce the potential demand of tasks. Tzur (2008) argued that teachers modify tasks when planning if they anticipate that students might not engage with the tasks without assistance. Charalambous (2008) argued that the mathematical knowledge of teachers is a factor in determining whether they reduce the mathematical demand of tasks based on their expectations for the students. Another factor that places pressure on teachers is the reluctance of some students to take risks in their learning. Desforges and Cockburn (1987), for example, reported a detailed study of primary classrooms in the United Kingdom and found that students and teachers conspired to reduce the level of risk for the students. Desforges and Cockburn argued that teachers can sometimes avoid the challenge of dealing with students who have given up by reducing the demand of the task rather than reflecting on what might be causing them to give up. Teachers who increase the challenge of tasks have not been so systematically studied, but strategies for doing so have been reported by Knott et al. (2013) (see above), Lee, and Lee and Park (2013) (see Chap. 5), Prestage and Perks (2007).

Of course, many of the decisions on how to introduce tasks are made during the process of the introduction itself. For example, teachers explore what prerequisite language is known by the students and what they understand about the context in which the task is being posed. Again involving teachers more closely in the intentions of the task designer, or the design process itself, may help to inform the task introduction process.

3.7.3 Access to Tasks by All Students

A third pedagogical anticipation is that if a task is appropriately challenging for most students, it can be anticipated that some will find it too difficult and may not engage with the task or rely too heavily on prompts from the teacher. The metaphor of Vygotsky's (1978) Zone of Proximal Development defines the work of the class as going beyond tasks that students can solve independently, so that the students are working on challenges for which they need support. It seems that one approach is for teachers to plan variations to the original task that are more accessible for those students experiencing difficulty or to plan tasks with multiple entry points, providing access for all students.

This notion of planned task variations is a consistent theme in advice to teachers. For example, a working group of teachers identified 34 different strategies they used when intervening while students are working (Association of Teachers of Mathematics (ATM), 1988). The strategies were then grouped under headings that describe the major decisions teachers have to make about interventions, such as whether or not to intervene, why intervention is advisable, how to initiate an intervention, whether to withdraw or proceed with the intervention, how to end an intervention, and so on. The level of detail was fine-grained; for example, there were 14 specific intervention suggestions about supporting students experiencing difficulty, about half of which relate to task differentiation. Although it makes no sense to assume that a teacher can adopt all such strategies successfully, articulating teachers' practices in this way does make them available for others to use.

Christiansen and Walther (1986), in describing the nature of student engagement in their learning, argued that:

> Through various means, actions are envisaged, discussed and developed in a co-operation between the teacher and the students. One of the many aims of the teacher is here to

> differentiate according to the different needs for support but to ensure that all learners recognise that these processes of actions are created deliberately and with specific purposes. (p. 261)

It is assumed that such approaches involve teachers inviting students who experience difficulty to work on tasks that are similar to the ones undertaken by other class members, but differentiated in some way to increase the accessibility of the task without reducing conceptual content. The design of these alternate tasks can be undertaken by the original designer or by the teacher, either in anticipation or interactively during the lesson.

It is perhaps in consideration and anticipation of students' responses to tasks that the teacher's role becomes critical. As described in this section, the teacher considers the motivation of the students, the level of prerequisite knowledge to engage with the task, the prevailing classroom mathematical culture, and the extent to which the task can be differentiated to allow all students to engage effectively.

3.8 Summary and Conclusion

This chapter described factors influencing task design and features of task design that inform and are informed by teachers' decisions about mathematical goals and anticipated pedagogies. By analyzing three typical tasks as examples, attention was paid to five dilemmas (context, language, structure, distribution, levels of interaction) and six tensions (epistemic, cognitive, interactional, mediational, affective, ecological). Designers and teachers need to consider these multiple dimensions to address different aspects of the task and pedagogic design, based on their anticipation of classroom implementation and students' learning.

The process of task design could focus on either or both the specialized and practical aspects of mathematics, formal and natural language, and this focus can be specific or implied. We recognized and described the central role of the teacher in design/redesign of tasks and their implementation.

Teachers and designers might be aware of the cultural assumptions of a task in their (re)design process. Especially in the implementation in the classroom, students' understanding and activities are influenced by social and sociomathematical norms, and it is necessary to consider what the culture of the classroom might bear and at least suggest what kind of classroom culture and instructional approach might appropriately support the implementation of the designed task or task sequence.

Designers may seek to either limit the decision-making of teachers or augment it, either as part of the design process or by direct collaboration. Teachers in turn anticipate the pedagogies through the creation of compatible classroom cultures and consideration of hypothetical learning trajectories. Both designers and teachers may consider affective issues of task design, including the motivational responses of students and the need to maximize the engagement of all students.

References

Askew, M., & Canty, L. (2013).Teachers and researchers collaborating to develop teaching through problem solving in primary mathematics. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 531–540), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Association of Teachers of Mathematics (ATM). (1988). *Reflections on teacher intervention*. Derby: ATM.

Barbosa, J. C., & de Oliveira, A. M. (2013). Collaborative groups and their conflicts in designing tasks. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 541–548), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Bartolini Bussi Maria, G., Canalini, R., & Ferri, F. (2011). Towards cultural analysis of content: problems with variation in primary school. In J. Novotna & H. Moraova (Eds.), *Proceedings of SEMT'11, International Symposium Elementary Maths Teaching: The mathematical knowledge needed for teaching in Elementary School* (pp. 9–20). Prague: Faculty of Education, Charles University.

Bartolini Bussi, M., Sun, X., & Ramploud, A. (2013). A dialogue between cultures about task design for primary school. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 549–558), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Brown, L., & Coles, A. (2013).On doing the same problem – first lessons and relentless consistency. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 617–626), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C.-P., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499–531.

Charalambous, C. Y. (2008). Mathematical knowledge for teaching and the unfolding of tasks in mathematics lessons: Integrating two lines of research. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda (Eds.), *Proceedings of the 32nd Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 281–288). Morelia: PME.

Christiansen, B., & Walther, G. (1986). Task and activity. In B. Christiansen, A. G. Howson, & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 243–307). Dordrecht, The Netherlands: Reidel.

Chu, H. (2013). Scaffolding tasks for the professional development of mathematics teachers of English language learners. In C. Margolinas (Ed.) *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 559–568), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Cobb, P., & McClain, K. (1999). Supporting teachers' learning in social and institutional contexts. In F.-L. Lin (Ed.), *Proceedings of the 1999 International Conference on Mathematics Teacher Education* (pp. 7–77). Taipei: National Taiwan Normal University.

Cooper, B., & Dunne, M. (1998). Anyone for tennis? Social class differences in children's responses to national curriculum mathematics testing. *The Sociological Review*, (Jan), 115–148.

Desforges, C., & Cockburn, A. (1987). *Understanding the mathematics teacher: A study of practice in first schools*. London: The Palmer Press.

Dweck, C. S. (2000). *Self theories: Their role in motivation, personality, and development*. Philadelphia, VA: Psychology Press.

Ernest, P. (1994). Varieties of constructivism: Their metaphors, epistemologies and pedagogical implications. *Hiroshima Journal of Mathematics Education, 2*, 1–14.

Ernest, P. (2010). The social outcomes of learning mathematics: Standard, unintended or visionary? In *Make it count: What research tells us about effective mathematics teaching and learning* (pp. 21–26). Camberwell: Australian Council for Educational Research.

Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. London: Routledge.

GEMAD. (2013). An experience of teacher education on task design in Colombia. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 569–578), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Giménez, J., Font, V., & Vanegas, Y. (2013). Designing professional tasks for didactical analysis as a research process. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 579–588), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Goos, M., Geiger, V., & Dole, S. (2010). Auditing the numeracy demands of the middle years curriculum. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, pp. 210–217). Fremantle: MERGA.

Goos, M., Geiger, V., & Dole, S. (2013).Designing rich numeracy tasks. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 589–598), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Gueudet, G., & Trouche, L. (2011). Communities, documents and professional geneses: Interrelated stories. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *Mathematics curriculum material and teacher development* (pp. 305–322). New York: Springer.

Hashimoto, Y., & Becker, J. (1999). The open approach to teaching mathematics – creating a culture of mathematics in the classroom: Japan. In L. Sheffield (Ed.), *Developing mathematically promising students* (pp. 101–120). Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., & Wearne, D. (1997). Instructional tasks, classroom discourse and student learning in second grade arithmetic. *American Educational Research Journal, 30*, 393–425.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Jaworski, B. (2014). Mathematics education development. Research in teaching Learning in practice. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the 37th Annual Conference of the Mathematics Education Research Group of Australasia, pp. 2–23). Sydney: MERGA.

Jaworski, B., Goodchild, S., Eriksen, S., & Daland, E. (2011). Inquiry, mediation and development: Use of tasks in developing mathematics learning and teaching. In O. Zaslavsky & P. Sullivan (Eds.), *Constructing knowledge for teaching secondary mathematics: Tasks to enhance prospective and practicing teacher learning* (pp. 143–160). Norwell, MA: Springer.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, D.C.: National Academy Press.

Knott, L., Olson, J., Adams, A., & Ely, R. (2013). Task design: Supporting teachers to independently create rich tasks. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 599–608), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Kullberg, A., Runesson, U., & Mårtensson, P. (2013). The same task? – Different learning possibilities. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 609–616), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Kullberg, A., Runesson, U., & Mårtensson, P. (2014). Different possibilities to learn from the same task. *PNA, 8*(4), 139–150.

Lee, K., Lee, E., & Park, M. (2013). Task modification and knowledge utilization by Korean prospective mathematics teachers. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 347–356), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*, 454–482.

Middleton, J. A. (1995). A study of intrinsic motivation in the mathematics classroom: A personal construct approach. *Journal for Research in Mathematics Education, 26*(3), 254–279.

Peled, I. (2008). Who is the boss? The roles of mathematics and reality in problem solving. In J. Vincent, R. Pierce, & J. Dowsey (Eds.), *Connected maths* (pp. 274–283). Melbourne: Mathematical Association of Victoria.

Peled, I., & Suzan, A. (2013). Designed to facilitate learning: Simple problems that run deep. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 633–640), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Prestage, S., & Perks, P. (2007). Developing teacher knowledge using a tool for creating tasks for the classroom. *Journal of Mathematics Teacher Education, 10*(4–6), 381–390.

Remillard, J. T., Herbel-Eisenmann, B. A., & Lloyd, G. M. (Eds.). (2009). *Mathematics teachers at work: Connecting curriculum materials and classroom instruction*. New York: Routledge.

Rollard, R. G. (2012). Synthesizing the evidence on classroom goal structures in middle and secondary schools: A meta analysis and narrative review. *Review of Educational Research, 82*(4), 396–435.

Ron, G., Zaslavsky, O. & Zodik, I. (2013). Engaging teachers in the web of considerations underlying the design of tasks that foster the need for new mathematical concept tools. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 641–647), Oxford, UK. Available from http://hal.archives-ouvertes.fr/hal-00834054

Sawatzki, C., & Sullivan, P. (2015). Situating mathematics in "real life" contexts: Using a financial dilemma to connect students to social and mathematical reasoning. *Mathematics Education Research Journal*.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 114–145.

Smith, M. S., & Stein, M. K. (2011). *Five practices for orchestrating productive mathematics discussions*. Reston, VA/Thousand Oaks, CA: National Council of Teachers of Mathematics/Corwin Press.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455–488.

Sullivan, P., Askew, M., Cheeseman, J., Clarke, D., Mornane, A., Roche, A., et al. (2014). Supporting teachers in structuring mathematics lessons involving challenging tasks. *Journal of Mathematics Teacher Education*, April. doi:10.1007/s10857-014-9279-2

Sullivan, P., Mousley, J., & Jorgensen, R. (2009). Tasks and pedagogies that facilitate mathematical problem solving. In B. Kaur (Ed.), *Mathematical problem solving* (pp. 17–42). Association of Mathematics Educators. Singapore/USA/UK: World Scientific Publishing.

Sullivan, P., Tobias, S., & McDonough, A. (2006). Perhaps the decision of some students not to engage in learning mathematics in school is deliberate. *Educational Studies in Mathematics, 62*, 81–99.

Tzur, R. (2008). Profound awareness of the learning paradox. In B. Jaworski & T. Wood (Eds.), *The mathematics teacher educator as a developing professional* (pp. 137–156). Sense: Rotterdam.

Vygotsky, V. (1978). *Mind in society*. Cambridge, MA: Harvard University Press.

Watson, A., & Sullivan, P. (2008). Teachers learning about tasks and lessons. In D. Tirosh & T. Wood (Eds.), *Tools and resources in mathematics teacher education* (pp. 109–135). Rotterdam: Sense Publishers.

Wiliam, D. (1998, July). *Open beginnings and open ends*. Paper distributed at the open-ended questions Discussion Group, International Conference for the Psychology of Mathematics Education, Stellenbosch, South Africa.