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Statics and buckling problems of aircraft structurally-anisotropic composite panels with the influence of production technology

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Abstract. The mathematical model relations for stress-strain state and for buckling investigation of structurally-anisotropic panels made of composite materials are presented. The mathematical model of stiffening rib being torsioned under one-side contact with the skin is refined. One takes into account the influence of panel production technology: residual thermal stresses and reinforcing fibers preliminary tension. The resolved eight order equation and natural boundary conditions are obtained with variation Lagrange procedure. Exact analytical solutions for edge problems are considered. Computer program package is developed using operating MATLAB environment. The influence of the structure parameters on the level of stresses, displacements, of critical buckling forces for bending and for torsion modes has analyzed.

1. Introduction

The paper discusses the stress-strain state and buckling problems of flat rectangular multiplied panel made of composite materials, the casing of which an eccentrically supported by the longitudinal-transverse stiffening set. The panel is subjected to arbitrarily distributed transverse loading in the stationary temperature field and to distributed loading applied to the edges in casing plane. Boundary conditions at the contour are assumed to be of quite general type.

One should take into consideration the technological factors occurring in the manufacture of composites: residual thermal stresses arising during cooling after hardening and pre-stressed tension in reinforcing fibers.

The theory of thin-walled elastic robs is used without the hypothesis of the zero shear deformation for reinforcing elements being in the complex resistance – two-plane bending and limited torsion. Further development of the theory of thin-walled elastic robs related to the contact problem for the skin and the rib with improved rib model reflects the scientific novelty of the research.

The schematization of the panel as structurally anisotropic has been proposed as a design model when stress-strain state and critical forces of total bending form of buckling were determined. For many-waved torsion buckling study one should use the generalized functions set.

The problem of determining stress-strain state of structurally anisotropic panels is reduced to the solution of the boundary value problem for equation of the eighth order in the partial derivatives in a rectangular field. The differential equations of eighth order in partial derivatives are resolving for the pre-critical stress-strained state and for the buckling problem. The solution in closed form is designed by unitary trigonometric series for the particular case of conformable boundary conditions on two opposite sides and by the method of uniform solutions for arbitrary non-conformable boundary conditions at the contour. We examine all possible variants of the boundary edges restrictions in relation to the connecting plane problem and the bending one.

Computer program package in MATLAB has been performed for multi-criterion optimization for the design of structurally-anisotropic composite panels of FA.

2. Main relations of mathematical model

The papers [1–15] describe statics problems of structurally-anisotropic composite panels but the mathematical model offered lower is seemed to be new one.

The geometrical and physical relations are assumed at each *k*-ply on the macro-level for the skin accordingly to Kirchgoff hypothesis.

Stress-strain state components of each *k*-ply of composite stringers are calculated taking into account plate and rib displacement and rotation angle equalities. The cross-section warping is put free here. Shear strain occurring because of the thin-walled bar torsion is connected with its rotation relative to chosen pole and also relative to bending centre in view of «pure» torsion theory additives [16].

The problem consists in finding the base surface displacements.

Further discussion concerns the- mathematical model with the proposal of the infinitesimal of normal stresses caused by bar bending in panel plane and bar cross-section warping. We'll consider ribs working in tension-compression, in out- of-plane plate bending and in torsion.

Composite panel equilibrium equations and natural boundary conditions are formed by the variation method using Lagrange concept as a result of the minimization of the whole potential energy of the system.

Equilibrium differential equation system is written in operator form with linear differential operators. By using symbol integration method, when, for example, the first and the second expressions are identically satisfied, it can be reduced to a single eight order linear non-uniform resolving partial differential equation for the potential function $\Phi(x, y)$, with the help of which all accounting values are expressed.

Natural boundary conditions permit to determine internal force factors relations. Both forces and moments depend on longitudinal and tangential skin plane displacements functions $u_0(x, y)$, $v_0(x, y)$, and on deflection function $w(x, y)$. So it shows that the problem is integral; in other words, it can't be divided into plane part and bending part.

If the structurally-anisotropic composite panel structure derived as a result of energy averaging becomes orthotropic, linear differential operator of the resolving equation and linear differential operators of the relations between symmetrical stress-strain state components and potential function $\Phi(x, y)$ contain only even numbered derivatives with respect to x and y. Oblique symmetrical components are determined with odd numbered derivatives:

$$L^{(8)}\Phi = q\,, \tag{1}$$

$$L^{(8)} = \frac{K_{80}}{a^8}\frac{\partial^8}{\partial x^8} + \frac{K_{62}}{a^6 b^2}\frac{\partial^8}{\partial x^6 \partial y^2} + \frac{K_{44}}{a^4 b^4}\frac{\partial^8}{\partial x^4 \partial y^4} + \frac{K_{26}}{a^2 b^6}\frac{\partial^8}{\partial x^2 \partial y^6} + \frac{K_{08}}{b^8}\frac{\partial^8}{\partial y^8}$$

Constants $K_{8-i,i}$, i=0, 2, 4, 6, 8 in (1) depend on geometrical and stiffness structure characteristics; $x = x/a$, $y = y/b$ – non-dimensional coordinates with reference to the panel half-length a and to its width b.

General boundary value problem in the rectangular field is specified by elastic fixity conditions if:

$$x = const \quad \left(\gamma_1 u_0 + \delta_1 N_x\right) = \left(\gamma_2 v_0 + \delta_2 N_{xy}\right) = \left(\gamma_3 w + \delta_3 \overline{Q}_x\right) = \left(\gamma_4 w'_x + \delta_4 M_x\right) = 0\,,$$

$$y = const \quad \left(\alpha_1 u_0 + \beta_1 N_{yx}\right) = \left(\alpha_2 v_0 + \beta_2 N_y\right) = \left(\alpha_3 w + \beta_3 \overline{Q}_y\right) = \left(\alpha_4 w'_y + \beta_4 M_y\right) = 0\,, \tag{2}$$

γ_i, δ_i, α_i, β_i , i = 1, 2, 3, 4 are from 0 to 1, which can be used to examine all possible boundary restraint combinations for connected plane and bending problems: free edge, simply supported edge, sliding fixation, constraining. In other words edge conditions may be kinematic, static and mixed.

Boundary value problems solution for (1) is constructed in a closed form by unitary Fourier series for particular boundary conditions along two opposite sides. These restrictions are called conformable one and they satisfy hinging condition about bending but sliding fixation condition in tangential direction for plane problem when the panel is loaded by shear force flows along its longitudinal edges. Face conditions are arbitrary and may be of elastic fixity one for symmetric or non-symmetric boundary value problems.

3. Statics problem results and discussion

As an example one calculated the displacements and stresses on the laminate macro-level in flat angle panel from carbon-plastic eccentrically stiffened in longitudinal direction. Edge conditions are symmetric by x coordinate. Normal loading $q(x, y) = const$.

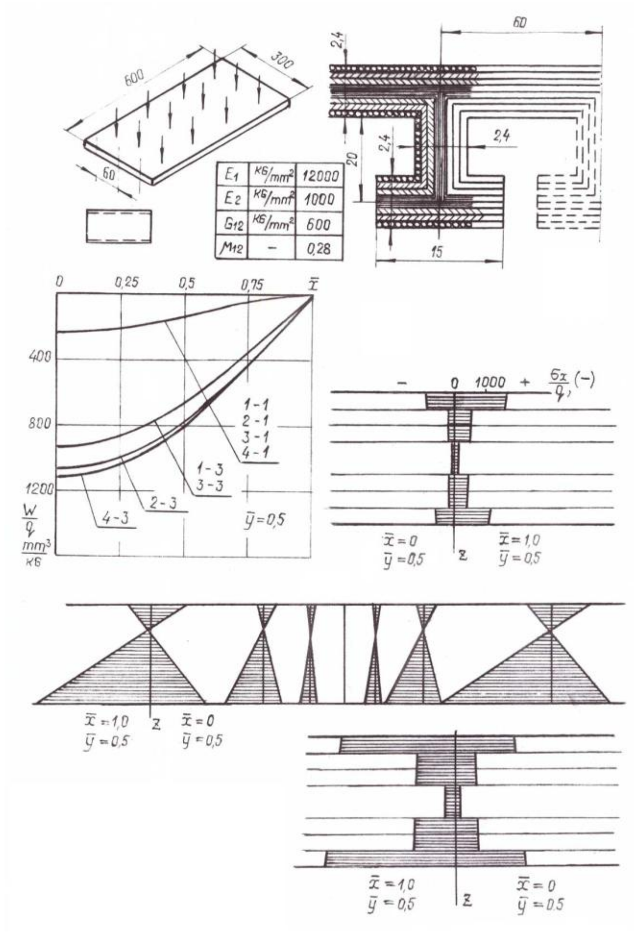
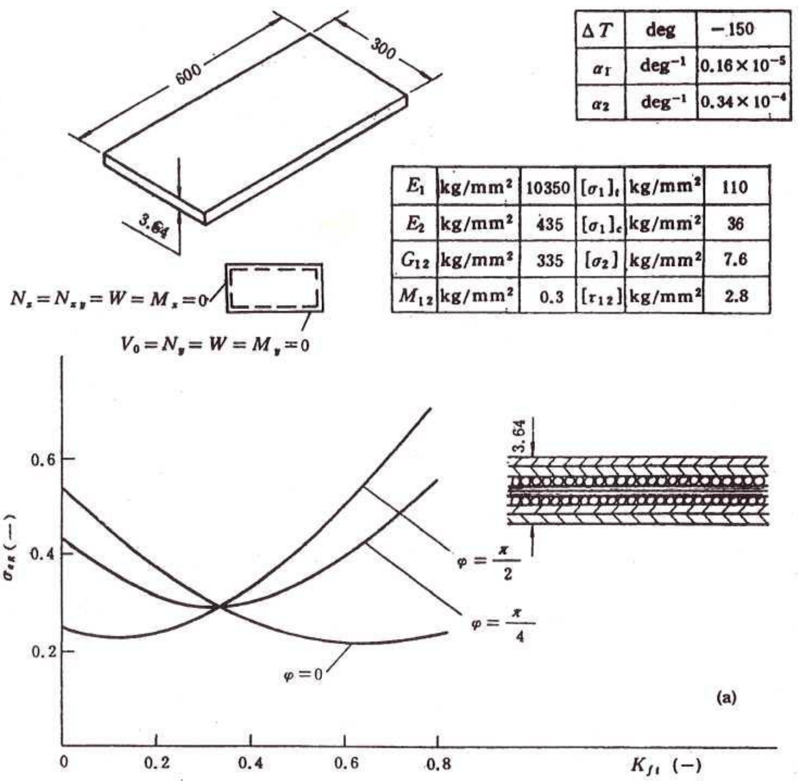


Figure 1. The panel, stiffened in longitudinal direction.
Deflections and normal stresses
Boundary conditions at lateral edges: plane problem – bending
1 – constraining, 2 – sliding fixation, 3 – simply supported edge, 4 – free edge

The deflections substantially consist on plane boundary conditions while lateral edges are simply supported about bending. When these edges are constrained for bending the deflection curves coincide practically while the restriction conditions about plane problem are varied.

As the next example we have calculated the residual thermal stresses brought about in cooling after hardening in flat angle panel made of carbon-epoxy having non-symmetric structure along its thickness (Figure 2). The panel contour is simply supported with respect to bending, and longitudinal sides restrictions correspond to the sliding fixation in tangential direction but lateral edges are free from forces and moments relative to plane problem.



ΔT	deg	−150
α_1	deg^{-1}	0.16×10^{-5}
α_2	deg^{-1}	0.34×10^{-4}

E_1	kg/mm^2	10350	$[\sigma_1]_t$	kg/mm^2	110
E_2	kg/mm^2	435	$[\sigma_1]_c$	kg/mm^2	36
G_{12}	kg/mm^2	335	$[\sigma_2]$	kg/mm^2	7.6
M_{12}	kg/mm^2	0.3	$[\tau_{12}]$	kg/mm^2	2.8

Figure 2. The panel with non-symmetric package structure.
The influence of the reinforcing fibers preliminary tension on the residual temperature stresses

The estimation of the influence of the reinforcing fibers preliminary tension on the residual thermal stresses level has been performed. For the composite panel made of longitudinal, lateral and also oblique plies (Figure 2) the residual thermal stresses decrease while the reinforcing fiber pre-stressed tension increases. There is the optimum level $K_{ft} = 0.35$ (35% allowable *k*-ply fiber deformation) when the whole package stresses are equal and minimum.

4. Buckling problems

The papers [17–25] describe buckling problems of structurally-anisotropic composite panels but the presented mathematical model is seemed to be new one.

The differential equations of eighth order in partial derivatives are the resolving ones for the pre-critical stress-strained state and for the buckling problems [26].

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The critical force definition of the general bending form of the thin-walled system buckling as well as the critical force definition of the many-waved torsion buckling are being of the most interest in accordance with traditional design practice. Both cases bending is integral with plane stress state so the buckling problem results in the value problem solving for the eight order partial derivatives equation in the rectangular field.

The solution is designed in closed form by double Fourier series for particular conformable boundary conditions along the panel contour. One can estimate the influence of production technology factors on the bearing strength of structurally-anisotropic composite panels if the before-critical stress state is taken into consideration or boundary conditions are non-conformable and the solution is formed by unitary trigonometric series or with the help of uniform solution method.

5. Buckling problem results and discussion

Figure 3 shows the influence of the composite panel sizes on the force level corresponding to general bending stability loss form and many-waved torsion buckling. The conditions of that or other buckling form existence are exposed.

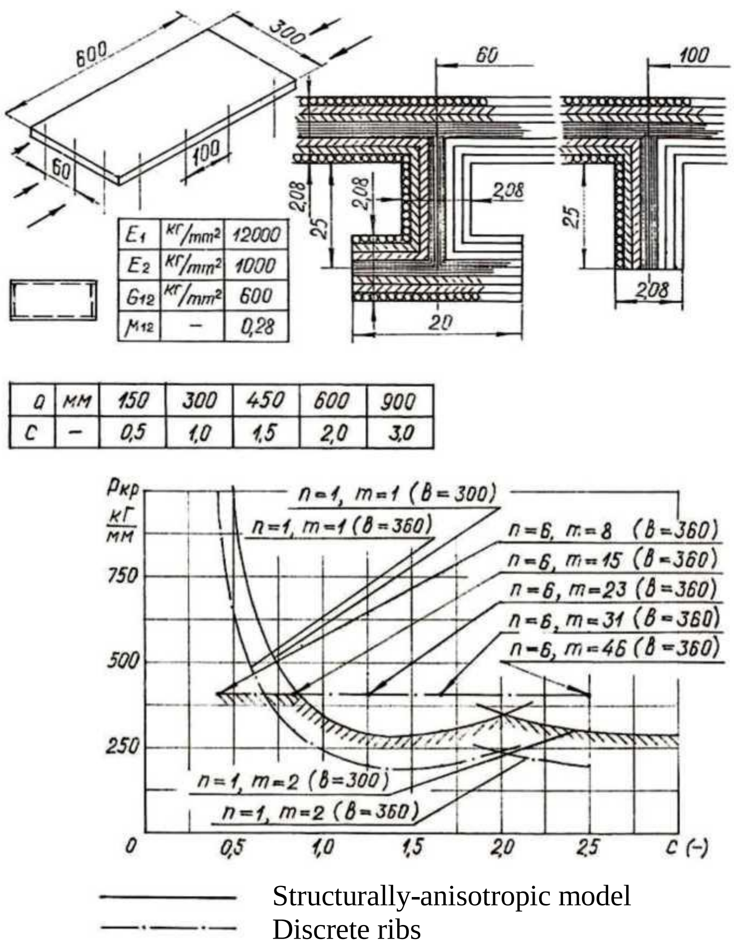

Figure 3. The panel with longitudinally-lateral stiffening set.
The influence of panel sizes on critical force level

For short panel if $c < 0{,}75$ there appears many-waved torsion buckling: $n = 6$, $m = 8$. If $c = 0{,}75$ the panel is equally buckled. The buckling for panels with $0{,}75 < c < 2{,}0$ has general bending form $n = 1$, $m = 1$, while $c > 2{,}0$ there is also general bending form, but $n = 1$, $m = 2$.

The testing series of uniform compressed stiffened composite panels carry to stability loss (Figure 4) have been made with the use of the special fixture.

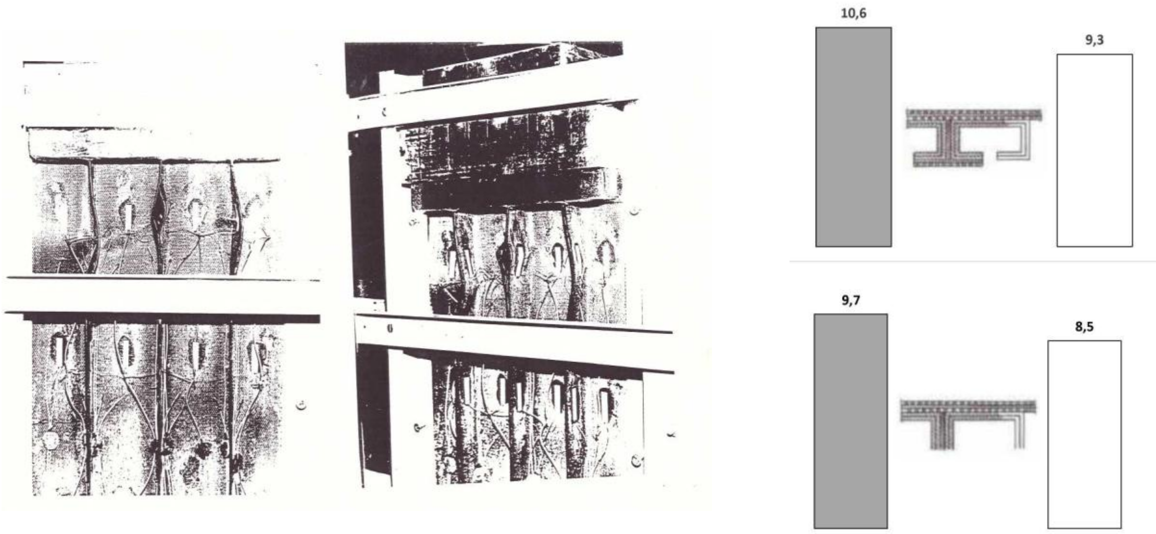

Figure 4. Experimental specimen

The refined theoretical results and the experimental data are in agreement qualitatively by buckling forms and quantitatively by critical stresses to the precision of 12% - 13%. It confirms the mathematical model authenticity.

6. Conclusion

Since we get the solution by analytical methods, the calculation time is of the minimum one. That is of interest from the point of view of practical design using parametric analysis. The results of the stress analysis calculations as well as the results of the buckling analysis calculations offer the opportunities for reducing and optimization of the aircraft elements weight characteristics.

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