Mathematical Social Sciences 114 (2021) 72–86

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

Inflation, endogenous quality increment, and economic growth

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ARTICLE INFO

Article history:
Received 20 March 2021
Received in revised form 26 September 2021
Accepted 5 October 2021
Available online 23 October 2021

Keywords:
Monetary policy
Economic growth
R&D
Endogenous quality increment

ABSTRACT

This study explores the effects of monetary policy in a Schumpeterian growth model with endogenous quality increment and distinct cash-in-advance (CIA) constraints on consumption, manufacturing and R&D investment. When the CIA constraint is only on consumption, an increase in the nominal interest rate may stifle economic growth by lowering the arrival rate of innovation and stimulate it at the same time by raising the size of quality increment. An additional CIA constraint on manufacturing weakens the growth-retarding effect and enhances the growth-promoting effect, whereas an additional CIA constraint on R&D strengthens only the negative growth effect. Our quantitative analysis finds that the relation between inflation and growth is generally hump-shaped, but the welfare effect of inflation is negative.

1. Introduction

In this study, we develop a Schumpeterian growth model to analyze the effects of monetary policy on the size of quality increment, economic growth, and social welfare, respectively. In contrast to the previous studies that assume an exogenous quality step size, this study extends the innovation-driven growth model by incorporating an endogenous quality increment channel through which monetary policy induces noticeable impacts on real variables. Money is introduced to this growth-theoretic framework by using the most generalized liquidity constraint via cash in advance (CIA). Specifically, in addition to the well-established approach of a CIA constraint on consumption as in Lucas (1980) and Dotsey and Sarte (2000), in this study we also consider a CIA constraint on manufacturing as in Fuerst (1992) and Arawatari et al. (2018), and a CIA constraint on R&D investment as in Chu and Cozzi (2014) and Chu et al. (2015).[1]

Our assumption regarding CIA constraints on firms' manufacturing and R&D investment is strongly motivated by recent empirical findings in firms' liquidity constraints. For example, Bates et al. (2009) and Lyandres and Palazzo (2016) find that the average cash-to-assets ratios for the US firms have sharply increased and become more than doubled since 1980. Ma et al. (2020) report a positive correlation between the industry-level cash- and R&D-to-assets ratios in the US. These results suggest a severe liquidity constraint on firms' behavior. Moreover, the empirical findings of Liu et al. (2008) indicate that firms' manufacturing activities are subject to cash constraint. More recent studies, such as Brown et al. (2012) and Brown and Petersen (2015), also reveal that firms tend to use cash to finance investment in R&D, the activities of which, however, suffer from liquidity constraint.

In this monetary Shumpeterian growth model augmented by different CIA constraints, we derive the following results. In the presence of a CIA constraint exclusively on consumption expenditure, an increase in the nominal interest rate raises the real wage rate through reducing labor supply, which generates two counteracting effects on economic growth. First, given that the price markup is increasing in the size of quality increment, a higher wage rate tends to decrease monopoly profit. To recoup a high profit flow, entrepreneurs are incentivized to pursue more radical innovations. Consequently, the increased size of quality increment causes the economic growth rate to rise. Second, a higher nominal interest rate discourages R&D incentives since entrepreneurs face a higher R&D cost in employing labor to produce inventions. As a result, the arrival rate of innovation decreases, causing the economic growth rate to decline. Since the economic growth rate is jointly determined by the arrival rate of innovation and the size of quality increment, the overall effect of the nominal interest rate on economic growth depends on the balance between the above competing forces. By calibrating the model to the US economy, we find that the relation between the nominal interest rate and economic growth is more likely to be monotonically decreasing. Conditional on the Fisher equation that predicts a positive *long-run* relation between the nominal interest

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[1] See Wang and Yip (1992) for a comparison of three reduced-form approaches of introducing money demand: the money-in-utility-function approach, the cash-in-advance approach, and the transaction-costs approach. We focus on the CIA approach mainly, because recent empirical studies support a liquidity constraint on firms' production and R&D activities.

https://doi.org/10.1016/j.mathsocsci.2021.10.001
0165-4896/

rate and the inflation rate (see Mishkin (1992) and Booth and Ciner (2001) for supportive empirical evidence), our model also implies a negative correlation between inflation and economic growth.

When CIA constraints on consumption and manufacturing are present, a rise in the nominal interest rate reinforces the aforementioned positive effect through causing a larger decline in the monopoly profit, whereas it weakens the negative effect through producing an additional reallocation effect that shifts labor employment from manufacturing to R&D. In this case, the nexus between inflation and economic growth can be either negative or hump-shaped, depending on the strength of the CIA constraint on manufacturing.[2] Our study thus provides a novel mechanism that potentially reconciles the mixed empirical evidence on the relation between inflation and economic growth. For example, Vaona (2012) and Barro (2013) find a monotonically decreasing inflation-growth relation. Nevertheless, a number of empirical studies, such as Khan and Senhadji (2001), Burdekin et al. (2004), and Eggoh and Khan (2014), have documented a non-monotonic relation instead.[3]

Furthermore, when consumption expenditure and R&D investment are constrained by cash, a higher nominal interest rate weakens the positive effect on the quality step size and strengthens the negative effect on the innovation arrival rate. This is because a higher nominal interest rate now leads to a larger increase in the R&D cost and thereby a larger decrease in the innovation arrival rate. In addition, the lowered R&D labor demand in turn suppresses the rise in the wage rate that is caused by a stronger constraint on consumption. This then depresses the positive impact of the nominal interest rate on the size of quality increment, as the decline in the monopoly profit becomes smaller in this circumstance. Therefore, the economic growth rate is monotonically decreasing in the nominal interest rate.

Our numerical analysis shows that, by calibrating the model to the US economy, inflation and growth can exhibit either a monotonically decreasing or an inverted-U relation, depending on which aforementioned CIA constraints are imposed. Interestingly, in all above cases, welfare is always decreasing in the nominal interest rate, implying that Friedman rule (i.e., zero-nominal-interest-rate targeting) is socially optimal. Finally, to test the empirical relevance of the model predictions, in Section 5, we perform an empirical analysis by using the US data from 1980 to 2020 and the result reveals a significantly inverted-U effect of inflation on the growth rate of GDP per capita in the US. Therefore, our model in the presence of (i) CIA constraints on consumption and manufacturing and (ii) CIA constraints on the three channels in total (i.e., consumption, manufacturing, and R&D) is able to adequately characterize this stylized fact.

This study closely relates to the literature on inflation and innovation. A representative along this line of studies is the pioneering work of Marquis and Reffett (1994), which explores the effects of inflation on growth in the framework of Romer (1990).[4] A great number of subsequent studies have analyzed the effects of inflation in a Schumpeterian quality-ladder model with an identical step size of quality improvement, such as Chu and Lai (2013), Chu and Cozzi (2014), Chu et al. (2015), Chu and Ji (2016), Huang et al. (2017), Oikawa and Ueda (2018), Huang et al. (2021), Gil and Iglésias (2020), and Zheng et al. (2019). One novel exception is Chu et al. (2017), who consider the heterogeneity of quality step sizes by assuming that the quality increment is drawn from an exogenously given distribution, instead of the endogenous choice by entrepreneurs. Our study complements their interesting study and contributes to the literature by allowing the step size of quality increment to be endogenously chosen by profit-maximizing entrepreneurs. Combined with the conventional frequency-of-innovation channel, the novel feature of endogenous quality step size provides a new mechanism to explain the (potentially) inverted-U relation between inflation and economic growth, which helps to reconcile the discrepancy in the empirical literature.

In addition, the positive relation between inflation and price markups in this model is consistent with the result in Wu and Zhang (2001) within a growth framework,[5] but it differs from the widely recognized implication of standard New Keynesian models featuring sticky prices. Due to mixed empirical evidence, however, the positive inflation-markup relation is not necessarily implausible. For example, Bils (1987), Rotemberg and Woodford (1991, 1999), Martins and Scarpetta (2002), and Gali et al. (2007) provide empirical evidence supportive of countercyclical markups; and Banerjee and Russell (2001) and Banerjee et al. (2001) identify a negative long-run relation between inflation and markup in Australia and most of the G7 countries. In sharp contrast, exploiting the Solow residual to estimate the cyclical movements in markups, Haskel et al. (1995) explore a panel data set of two-digit UK manufacturing industries, and find evidence for strongly procyclical markups. Using both aggregate and detailed manufacturing industry data, Nekarda and Ramey (2013) suggest that markups are procyclical unconditionally, and either mildly procyclical or acyclical conditional on demand shocks. Using detailed micro data on local house prices, retail prices and households shopping intensity, Stroebel and Vavra (2019) show that rising house prices increase consumers' demand by reducing their sensitivity to price changes, and firms raise markups in response. Their novel evidence suggests a procyclical desired or natural markup, which responds to monetary policy endogenously.[6] In fact, recent empirical evidence has motivated macroeconomic theorists to reinvestigate existing general equilibrium models for a better understanding of the mechanism under which a positive relation between inflation and price markups can be shaped.[7] This study exploits the Schumpeterian growth model and provides a discussion on an alternative possible channel inducing a positive inflation-markup correlation.

The rest of this study is organized as follows. Section 2 presents the model. Sections 3 and 4 analytically and numerically explore the effects of monetary policy on the quality increment, economic growth, and social welfare, respectively. Section 5 conducts an empirical analysis. Section 6 concludes.

2 In a more general case in which consumption expenditure, manufacturing, and R&D investment are all constrained by cash, a higher nominal interest rate raises the quality step size, decreases the innovation arrival rate, and finally causes a hump-shaped impact on economic growth.

3 Earlier studies indeed find a negative relation between steady inflation and growth across countries, such as Cooley and Hansen (1989), whereas later works, starting from Sarel (1996) and Ahmed and Rogers (2000), generally find a positive correlation in low-inflation industrialized economies. See López-Villavicencio and Mignon (2011) for a more comprehensive review on the linkage between inflation and economic growth.

4 Hori (2017) and Arawatari et al. (2018) also consider monetary policy in the Romer variety-expansion model with heterogeneity in the productivity of R&D entrepreneurs.

5 Wu and Zhang (2001) develop a neoclassical growth model with endogenous price markup, which is determined by firm number and firm size, and predict a positive linkage between inflation and markup.

6 Desired or natural markup is defined as the markup under perfectly flexible prices. See Nekarda and Ramey (2013) for a detailed survey of the literature on the cyclicality of price markups

7 For example, Phaneuf et al. (2018) propose a general equilibrium model with purely forward-looking price setters, and show that, in the existence of working capital financing, marginal cost can be directly affected by the nominal interest rate, the mechanism of which is able to induce procyclical movements in price markups.

2. Model

In this section, we present the monetary Schumpeterian growth model featuring quality increment that is endogenously chosen by optimizing entrepreneurs. The framework is based on the classical quality-ladder growth model in Grossman and Helpman (1991). We introduce money demand via CIA constraints on consumption as in Lucas (1980) and constraints on manufacturing as in Fuerst (1992) and Arawatari et al. (2018). The nominal interest rate serves as the monetary policy instrument, and the effects of monetary policy are examined by considering the implications of altering the rate of nominal interest on quality increment, innovation and economic growth, respectively.

2.1. Household

Consider an economy with a representative household whose intertemporal preference is given by

$$U = \int_0^{\infty} e^{-\rho t}[\ln c_t + \theta \ln(1-L_t)]dt, \tag{1}$$

where c_t is the consumption of final good and L_t is the supply of labor. The parameters $\rho > 0$ and $\theta \geq 0$ represent, respectively, the subjective discount factor and leisure preference. We assume that the size of household N_t does not grow over time and equals N_0 at time $t = 0$, which is normalized to unity.[8]

We choose the final good to be the numeraire. Thus, the household's budget constraint is given by

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_t L_t - \pi_t m_t - c_t + \tau_t, \tag{2}$$

where a_t is the real value of assets and the return rate of assets is the real interest rate r_t. w_t is the real wage rate. m_t is the real money balance held by the household and π_t is the inflation rate determining the cost of money holding. The household also receives a lump-sum transfer τ_t from the government. We assume that real money balances are required prior to purchasing the consumption good. The CIA constraint on consumption is $\xi c_t \leq m_t$, where $\xi > 0$ measures the strength of the CIA constraint.

The household maximizes her utility subject to the budget constraint and the CIA constraint. From standard dynamic optimization, we derive the following no-arbitrage condition:

$$\frac{\zeta_t}{\eta_t} - \pi_t = r_t, \tag{3}$$

where η_t and ζ_t are the Hamiltonian co-state variables on the budget constraint and the CIA constraint, respectively. As addressed by Bond et al. (1996) and Chang et al. (2019), this no-arbitrage condition states that the real rate of return on money (i.e., $\zeta_t/\eta_t - \pi_t$) must equal to the real rate of return on asset (i.e., r_t). With this no-arbitrage condition, we can derive the familiar Euler equation such that

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{4}$$

Moreover, we derive the optimality condition for labor supply such that

$$w_t(1-L_t) = \theta c_t(1+\xi i_t), \tag{5}$$

where $i_t = r_t + \pi_t$ is the nominal interest rate.

[8] By this assumption, we sidestep the issue of scale effects for analytical tractability. Alternatively, Peretto (1998), Segerstrom (1998), and Howitt (1999) provide important approaches that remove scale effects in the Schumpeterian growth model.

2.2. Production

There is a mass of competitive firms producing a unique final good by aggregating intermediate inputs according to the following Cobb–Douglas function:

$$y_t = \exp\left[\int_0^1 \ln x_t(j)dj\right], \tag{6}$$

where $x_t(j)$ is the quantity of intermediate goods in industry $j \in [0, 1]$. The final-good production function in (6) yields a unit-elastic demand with respect to each variety such that

$$x_t(j) = y_t/p_t(j), \tag{7}$$

where $p_t(j)$ denotes the price of $x_t(j)$.

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily occupied by an industry leader until the arrival of next innovation. We follow Peretto and Connolly (2007) and Arawatari et al. (2018) to assume that a fixed operating cost is required in production. Accordingly, the production function for the leader in industry j is

$$x_t(j) = \lambda^{n_t(j)}\left[L_{x,t}(j) - \kappa\right], \tag{8}$$

where $\lambda > 1$ is the quality increment of an innovation, $n_t(j)$ is the number of innovations that have occurred in industry j as of time t, $L_{x,t}(j)$ is the production labor in industry j, and $\kappa > 0$ is the fixed operating cost. We assume that monopolists need to borrow cash to facilitate production. Therefore, given $\lambda^{n_t(j)}$, the marginal cost of production for the leader in industry j is $mc_t(j) = w_t(1+\alpha i_t)/\lambda^{n_t(j)}$, where $(1+\alpha i_t)$ represents the additional cost due to a CIA constraint on manufacturing and $\alpha \in [0, 1]$ is the strength of the CIA constraint. Furthermore, we assume that the previous quality leader in industry j who owns the second-latest production technology is able to produce the same product $x_t(j)$ at a higher marginal cost of $(1+\alpha i_t)w_t/\lambda^{n_t(j)-1}$. Therefore, Bertrand competition implies that the profit-maximizing price $p_t(j)$ is given by

$$p_t(j) = \lambda mc_t(j),$$

which allows the current leader to exclude the competition of the previous leader.[9] Then the monopoly profit in industry j is

$$\Pi_t(j) = p_t(j)x_t(j) - w_t L_{x,t}(j)(1+\alpha i_t) = \left(\frac{\lambda-1}{\lambda}\right)y_t - \kappa w_t(1+\alpha i_t), \tag{9}$$

where we have applied (7) and (8). In addition, the demand function of manufacturing labor is

$$L_{x,t}(j) = \kappa + \frac{p_t(j)x_t(j)/[w_t(1+\alpha i_t)]}{\lambda} = \kappa + \frac{y_t/w_t}{\lambda(1+\alpha i_t)}, \tag{10}$$

where the second equality again applies (7). This equation implies that the demand of manufacturing labor is identical across industries.

2.3. Innovation

Denote by $v_t(j, \lambda)$ the value of the monopolistic firm in industry j that attempts to create an invention with a quality size of

[9] We assume that the previous leader is inactive when her profit is zero.

λ. Eq. (9) implies that the profit flow of each monopolist across industries $j \in [0, 1]$ is identical such that $v_t(j, \lambda) = v_t(\lambda)$ in a symmetric equilibrium.[10] Then the no-arbitrage condition for v_t is

$$r_t v_t = \Pi_t + \dot{v}_t - \mu_t v_t, \tag{11}$$

where μ_t is the aggregate intensity of research targeting at a state-of-the-art product and also the arrival rate of next innovation. Intuitively, the value $r_t v_t$ is equal to the sum of the profit flow Π_t, the potential capital gain $\dot{v}_t$, and the expected loss $\mu_t v_t$ due to creative destruction.

There is a unit continuum of entrepreneurs who employ R&D labor for innovation. Suppose that an entrepreneur $\omega \in [0, 1]$ who undertakes at intensity $\mu_t(\omega)$ for a time interval of length dt achieves success with a probability of $\mu_t(\omega)dt$. We assume that the resource cost of research effort depends on the size of the innovation that the entrepreneur pursues. In particular, research at intensity $\mu_t(\omega)$ requires $\mu_t(\omega)f(\lambda)$ units of labor, where $f'(\lambda) > 0$ and $f''(\lambda) > 0$. For tractability, we assume $f(\lambda) = \beta\lambda^{\epsilon}$, where $\beta > 0$ is a parameter and $\epsilon \equiv \lambda f'(\lambda)/f(\lambda) > 1$ is the elasticity of the resource requirement with respect to the size of attempted innovation. The R&D cost is thus given by $\mu_t(\omega)f(\lambda)w_t$. The entrepreneur ω chooses λ and $\mu_t(\omega)$ at every moment to maximize her expected profit such that[11]

$$\max_{\{\lambda, \mu_t(\omega)\}} \mu_t(\omega)v_t(\lambda)dt - \mu_t(\omega)f(\lambda)w_t dt.$$

The optimal choice of quality increment satisfies the following first-order condition:

$$v_t'(\lambda) = f'(\lambda)w_t. \tag{12}$$

which equates the marginal benefit of a larger innovation to the marginal cost of achieving it. The maximization of net benefits from R&D with respect to the choice of research intensity yields the zero-expected-profit condition such that

$$v_t(\lambda) = f(\lambda)w_t. \tag{13}$$

Moreover, in equilibrium, the unit measure of entrepreneurs implies that the aggregate research intensity (i.e., the innovation rate) is equal to the counterpart at the individual level, namely, $\mu_t \equiv \int_0^1 \mu_t(\omega)d\omega$.

2.4. Monetary authority

The monetary sector is formulated as in Arawatari et al. (2018). The monetary authority controls the nominal interest rate i, which is kept constant over time such that $i_t = i > 0$ for all time $t > 0$. The seigniorage revenue is rebated to the household via a lump-sum transfer. Denote by M_t the nominal money supply at time t. Thus, the budget constraint is given by $\tau_t = \dot{M}_t/P_t$, where P_t is the nominal price of the final good.

2.5. Decentralized equilibrium

Definition 1. The decentralized equilibrium consists of a sequence of prices $\{P_t, w_t, r_t, i_t, p_t(j), v_t\}_{t=0}^{\infty}$ and allocations $\{c_t, a_t, m_t, y_t, L_t, L_{x,t}, L_{r,t}\}_{t=0}^{\infty}$ such that the representative household maximizes utility taking $\{r_t, w_t\}$ as given; competitive final-good firms produce $\{y_t\}$ to maximize profits taking $\{p_t(j)\}$ as given; each differentiated intermediate-good producer j produces $x_t(j)$ and chooses $\{L_{x,t}(j), p_t(j)\}$ to maximize profits taking $\{w_t\}$ as given; entrepreneurs choose $\{\mu_t, \lambda\}$ to maximize expected profits taking $\{w_t\}$ as given; and all markets clear. That is, the final-good and asset markets clear such that $c_t = y_t$ and $a_t = v_t$, respectively, where v_t is the aggregate firm value. The labor-market-clearing condition is

$$L_{x,t} + L_{r,t} = L_t, \tag{14}$$

where $L_{x,t} \equiv \int_0^1 L_{x,t}(j)dj$ and $L_{r,t} = \int_0^1 \mu_t(\omega)f(\lambda)d\omega = \mu_t f(\lambda)$ are the aggregate demand of manufacturing labor and R&D labor, respectively.

Then we obtain the following result.

Lemma 1. *Holding constant the nominal interest rate i, the economy immediately jumps to a unique and stable balanced growth path along which each variable grows at a constant (possibly zero) rate.*

Proof. See Appendix A. □

In the steady state, the firm value v_t grows at the same rate as consumption and final goods do, and labor allocations are stationary. Applying the Euler Eq. (4) and the no-arbitrage condition (11), we can obtain the steady-state value of innovation such that

$$v_t(\lambda) = \frac{\Pi_t}{\rho + \mu}. \tag{15}$$

Now $v_t'(\lambda)$ can be calculated by using (15). Substituting $v_t'(\lambda)$ and (15) into the two first-order conditions for each entrepreneur (i.e., (12) and (13)), we have

$$\frac{v_t'(\lambda)}{v_t(\lambda)} = \frac{f'(\lambda)}{f(\lambda)} \Leftrightarrow \lambda = \frac{1 + 1/\epsilon}{1 - \kappa w_t(1 + \alpha i)/y_t}. \tag{16}$$

Notice that each entrepreneur takes the aggregate research intensity μ as given.

By substituting (9) and (15) into (13), the steady-state ratio of output to wage is given by

$$\frac{y_t}{w_t} = \frac{(\rho + \mu)f(\lambda) + \kappa(1 + \alpha i)}{(\lambda - 1)/\lambda}. \tag{17}$$

Substituting (17) into (10), together with the fact that $L_{x,t} = L_{x,t}(j)$, yields the aggregate demand of manufacturing labor such that

$$L_x = \kappa + \frac{(\rho + \mu)f(\lambda) + \kappa(1 + \alpha i)}{(\lambda - 1)(1 + \alpha i)}. \tag{18}$$

Moreover, using (5) and (17), we can rewrite the aggregate labor supply as

$$L = 1 - \theta(1+\xi i)\frac{c_t}{w_t} = 1 - \theta(1+\xi i)\frac{(\rho + \mu)f(\lambda) + \kappa(1 + \alpha i)}{(\lambda - 1)/\lambda}, \tag{19}$$

where the final-good resource condition has been applied. Next, substituting (18) and (19) into the labor-market-clearing

[10] See, for example, Cozzi et al. (2007) for a theoretical justification for the symmetric equilibrium in this strand of Schumpeterian growth model.

[11] It is useful to note that we adopt a two-step method solving the R&D firms' optimization problem. First, a typical R&D firm maximizes its discounted sum of profit streams by controlling the output level in each moment, taking as given the step size of innovation, λ. The solution satisfies the no-arbitrage condition given by (11). Next, the firm solves the one-shot problem by selecting λ to obtain the first-order condition (12). Such a two-step method works because the economy always stays on the balanced-growth equilibrium, as shown in Lemma 1. We thank the referee for this point.

condition (14) yields the following equation:

$$\kappa + \mu f(\lambda) + \frac{f(\lambda)(\rho+\mu)+\kappa(1+\alpha i)}{(\lambda-1)(1+\alpha i)} + \theta(1+\xi i)\frac{(\rho+\mu)f(\lambda)+\kappa(1+\alpha i)}{(\lambda-1)/\lambda} = 1$$
$$\Leftrightarrow \mu = \frac{\frac{(1-\kappa)(\lambda-1)(1+\alpha i)}{f(\lambda)} - \left[\frac{\kappa(1+\alpha i)}{f(\lambda)}+\rho\right][1+\theta\lambda(1+\xi i)(1+\alpha i)]}{1+(\lambda-1)(1+\alpha i)+\lambda\theta(1+\xi i)(1+\alpha i)}, \tag{20}$$

which contains two endogenous variables $\{\lambda, \mu\}$. In addition, using (16) and (17), we obtain the other equation that contains the endogenous variables $\{\lambda, \mu\}$ such that

$$\lambda - \frac{\kappa(1+\alpha i)(\lambda-1)}{f(\lambda)(\rho+\mu)+\kappa(1+\alpha i)} = 1+1/\epsilon \Leftrightarrow \mu = \frac{\kappa(1+\alpha i)/\epsilon}{(\lambda-1-1/\epsilon)f(\lambda)} - \rho. \tag{21}$$

Consequently, Eqs. (20) and (21) are the equations that solve the steady-state equilibrium of this model. In the following analysis, Eq. (20) is denoted as the "labor condition", whereas Eq. (21) is denoted as the "R&D condition".

Given the equilibrium innovation arrival rate and size of quality increment, we rewrite the production function of final goods by substituting (8) into (6) such that

$$y_t = Q_t L_x. \tag{22}$$

In this equation, Q_t is the aggregate technology level and defined as

$$Q_t = \exp\left(\int_0^1 n_t(j)dj \ln\lambda\right) = \exp\left(\int_0^t \mu_s ds \ln\lambda\right),$$

where the second equality applies the law of large number. Accordingly, the steady-state growth rate of technology (and also of final goods) is given by

$$g \equiv \frac{\dot{Q}_t}{Q_t} = \frac{\dot{y}_t}{y_t} = \mu^* \ln\lambda^*, \tag{23}$$

where μ^* and λ^* are the equilibrium values of research intensity and quality increment, respectively.

Before closing this section, we show that our analysis on how the nominal interest rate relates to quality increment, economic growth, and social welfare, also applies to the counterpart on how inflation relates to those variables, as justified in Chu and Cozzi (2014) and Chu et al. (2017). To see this, we combine the Fisher equation and the Euler equation to show that the inflation rate is given by $\pi = i - r = i - g(i) - \rho$. As long as $\partial g(i)/\partial i < 1$, we have $\partial\pi/\partial i = 1 - \partial g(i)/\partial i > 0$.[12] This positive *long-run* relation between the inflation rate and the nominal interest rate is also supported by the empirical evidence in Mishkin (1992) and Booth and Ciner (2001).

3. Implications of monetary policy

In this section, we analyze the effects of monetary policy on the optimal size of quality increment, innovation, economic growth, and social welfare, respectively. In Section 3.1, we consider a special case in which the CIA constraint is only on consumption. In Section 3.2, we consider the general case of both CIA constraints on consumption and manufacturing. In the next section (i.e., Section 4), we numerically evaluate the impacts of monetary policy on the aforementioned variables.

12 Under our calibrated parameter values, steady-state inflation is increasing in the nominal interest rate.

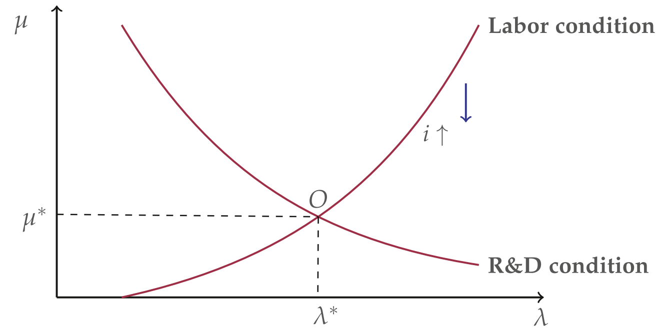

Fig. 1. The steady-state equilibrium under a CIA constraint on consumption.

3.1. CIA constraint on consumption

To better understand how monetary policy affects the real aspects, we first consider the special case where CIA constraint is exclusively on consumption. When manufacturing activities are not constrained by cash, which can be obtained by setting $\alpha = 0$, (20) and (21) are reduced to

$$\mu = \frac{\frac{(1-\kappa)(\lambda-1)}{f(\lambda)} - \left[\frac{\kappa}{f(\lambda)}+\rho\right][1+\theta\lambda(1+\xi i)]}{1+(\lambda-1)+\lambda\theta(1+\xi i)}, \tag{24}$$

and

$$\mu = \frac{\kappa/\epsilon}{(\lambda-1-1/\epsilon)f(\lambda)} - \rho, \tag{25}$$

respectively. Eq. (24) features a positive slope and a positive λ—intercept in the $\{\lambda, \mu\}$ space as shown in Fig. 1 by the labor-condition curve. In addition, Eq. (25) also contains two endogenous variables $\{\mu, \lambda\}$ but features a negative slope, with no intercepts, in the $\{\lambda, \mu\}$ space as shown in Fig. 1 by the R&D-condition curve. The intersection at point O in Fig. 1 determines the unique steady-state values for μ and λ.[13]

Fig. 1 shows that an increase in the nominal interest rate shifts down the labor-condition curve and leaves the R&D-condition curve unaffected, leading to a lower innovation rate accompanied by a larger size of quality increment. Intuitively, due to the CIA constraint on consumption, Eq. (5) shows that a higher nominal interest rate raises the opportunity cost of consumption, causing households to substitute for leisure. As a consequence, the decline in labor supply raises the real wage rate, yielding two opposing effects on economic growth. On the one hand, the rise in the wage rate decreases the monopoly profit flow for a given size of quality increment, as shown in (9). This in turn induces entrepreneurs to pursue a more radical innovation with a higher innovating firm value. On the other hand, the rise in the wage rate increases the R&D cost, which discourages the R&D incentive and thus reduces the innovation rate. Moreover, an attempt of a more radical innovation is associated with a higher demand in R&D labor and a larger R&D cost, both of which reinforce the negative impact of a rise in the nominal interest rate on the innovation arrival rate. The above results are summarized in the following Proposition 1.

Proposition 1. *Under the endogenous quality step size λ^*, a higher nominal interest rate decreases the arrival rate of innovation but increases the size of quality increment.*

Proof. Proven in the text. □

13 See Appendix A.2 for the details for which the intersection between the labor condition (24) and the R&D condition (25) is unique.

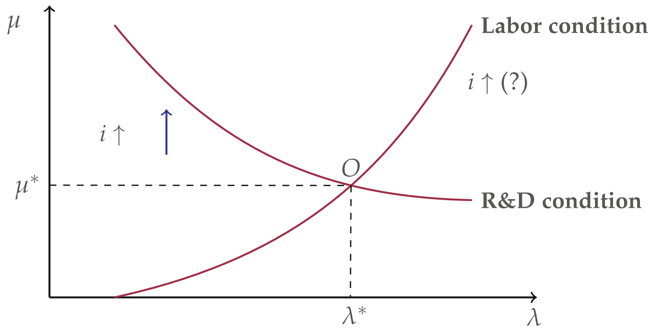

Fig. 2. The steady-state equilibrium under CIA constraints on consumption and manufacturing.

Differentiating (23) with respect to the nominal interest rate i yields

$$\frac{\partial g}{\partial i}=\underbrace{\frac{\partial \mu^*}{\partial i}}_{<0}\cdot\ln\lambda^*+\underbrace{\frac{\partial \lambda^*}{\partial i}}_{>0}\cdot\frac{\mu^*}{\lambda^*}.$$

In an economy in which the quality increment is exogenously given, the channel of changing the quality increment size through which monetary policy affects economic growth is shut down, i.e., $\partial\lambda^*/\partial i=0$. In this case, the economic growth rate g is a decreasing function of the nominal interest rate i, as in the existing studies such as Chu and Cozzi (2014). Nevertheless, in the economy in which the quality increment can be endogenously determined by entrepreneurs, varying the nominal interest rate can affect the economic growth rate through the size of quality increment in addition to the frequency of innovation. This is the novel mechanism in our model that could cause a non-monotonic effect of the nominal interest rate on the economic growth rate.

3.2. CIA constraints on consumption and manufacturing

We now proceed to the general case with CIA constraints on consumption and manufacturing. Fig. 2 describes the effects of a higher nominal interest rate on the quality step size and the innovation arrival rate. Comparing (20) and (21) to (24) and (25), it is obvious that the presence of an additional CIA constraint on manufacturing causes the R&D-condition curve to shift upward, but leads to an ambiguous impact on the labor-condition curve.

In this case, the overall impact of a higher nominal interest rate on the quality increment and innovation becomes ambiguous. The intuition for this result is as follows. Recall that the nominal interest rate raises the real wage rate through the channel of CIA on consumption. On the one hand, with a higher nominal interest rate, imposing a CIA constraint on manufacturing further reduces the monopoly profit, which reinforces the negative effect from the rising wage rate. This effect motivates entrepreneurs to pursue an even more radical innovation aiming to set a larger price markup and gain a higher profit flow. On the other hand, a CIA constraint on manufacturing creates an incentive for labor reallocation from the manufacturing sector to the R&D sector, which mitigates the negative effect of inflation on R&D from the rising wage rate. Whether a higher nominal interest rate increases or decreases the quality increment and innovation depends on the relative magnitude of the above effects. Given this ambiguity, we provide a discussion in the numerical analysis that follows.

Table 1
Parameter values and targeted moments.

Parameters		Targeted moments	
ρ	0.02	Innovation arrival rate	8%
ξ	0.17	M1-consumption ratio	0.17
α	1	Economic growth rate	2%
κ	0.0223	Time of employment	1/3
θ	1.8146	Average inflation rate	2.5%
β	0.1622		

4. Quantitative analysis

In this section, we calibrate the model to the US data and numerically evaluate the effects of the nominal interest rate (and the inflation rate) on quality increment, innovation, economic growth and social welfare, respectively. To facilitate the analysis, we assume $f(\lambda)=\beta\lambda^5$ as the benchmark functional form and consider alternative functions in the sensitivity analysis.[14] To perform this quantitative analysis, we assign steady-state values to the structural parameters $\{\rho,\xi,\alpha,\theta,\kappa,\beta\}$. The discount rate ρ is set to a conventional value of 0.02. As for the strength of the CIA constraint on consumption (i.e., ξ), we follow Zheng et al. (2019) to set it to 0.17, in order to match the ratio of M1-consumption in the US. As for the strength of the CIA constraint on manufacturing, we follow Arawatari et al. (2018) to set $\alpha=1$ as the benchmark. To pin down the value of the remaining parameters, we match the following long-run empirical moments. (a) Given that the conventional value of the economic growth rate is 2% and the long-run average inflation rate in the US is about $\pi=2.5\%$, the steady-state rate of nominal interest is determined by the Fisher equation such that $i=r+\pi=\rho+g+\pi=6.5\%$; (b) The standard time of employment to 1/3; (c) The arrival rate of innovation μ^* is set to an empirically relevant value of 8% as the benchmark.[15] Table 1 summarizes these moments and calibrated parameter values.

4.1. Results

Given the benchmark estimated parameters, we now quantify the impacts of the nominal interest rate (and the inflation rate) on the quality increment, the innovation rate, the economic growth rate, and the social welfare, respectively. Figs. 3(a) and 3(b) display that the size of quality increment is increasing in the inflation rate, but the arrival rate of innovation is decreasing in it. When raising the inflation rate from −0.0400 (i.e., $i=0$) to 0.1601 (i.e., $i=0.2$), the quality step size rises from 1.2795 to 1.2937, whereas the arrival rate of innovation declines from 0.0810 to 0.0773. As a result, the growth rate of output becomes an inverted-U function of the inflation rate. Fig. 4(a) shows that the growth-maximizing inflation rate is around 3.87%, which is consistent with the estimates in a number of empirical evidence such as Burdekin et al. (2004) and Kremer et al. (2013). This result indicates that the positive effect of inflation on the

[14] Assuming $f(\lambda)=\beta\lambda^5$ means that the elasticity is $\epsilon=5$. According to (21), $\lambda>1+1/\epsilon=1.2$ must hold. As shown below, given the conventional economic growth rate and arrival rate of innovation, the benchmark quality step size, namely the price markup, is 1.284. This result is consistent with the empirical evidence, since the market value of price markup is generally lower than 1.4 (see, for example, Jones and Williams (2000)). We also consider a sensitivity analysis on the function form of $f(\lambda)$ in Section 4.2.

[15] The existing literature has considered different values for the innovation arrival rate. For example, using a structural model to estimate, Caballero and Jaffe (2002) report an innovation rate of 4%. Laitner and Stolyarov (2013) find the roughly same value (i.e., 3.5%), whereas Lanjouw (1998) shows that the probability of obsolescence lies in the range of 7%–12%. We thus select an intermediate value of the above estimates in this exercise.

quality increment dominates the negative effect of inflation on the innovation arrival rate when the inflation rate is at a low level, and the positive effect is dominated by the negative one when the inflation rate becomes sufficiently high.

To explore the welfare effect of inflation, we derive the steady-state welfare function. This is obtained by imposing balanced growth on (1), which yields

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g}{\rho}\right) = \frac{1}{\rho}\left(\ln Q_0 + \ln L_x + \frac{g}{\rho}\right), \tag{26}$$

where Q_0 is normalized to unity, and L_x and $g = \mu \ln \lambda$ are given by (18) and (23), respectively. Fig. 4(b) shows that the welfare level is decreasing in the inflation rate. For example, raising the inflation rate from −0.0400 to 0.1601 causes the social welfare U to decline from −9.7924 to −17.8742. This result implies that Friedman rule (i.e., the nominal interest rate at the zero level) is optimal in this case.

4.2. Robustness analysis

In this subsection, we conduct two experiments to examine the extent to which the quantitative results would change: one is to reduce the strength of the CIA constraint on manufacturing to zero, and the other is to consider an alternative function of $f(\lambda) = \beta\lambda^3$.

We first consider the case of the CIA constraint only on consumption (i.e., $\alpha = 0$). Keeping other parameter values unchanged as in the benchmark, we evaluate the impacts of inflation on the interested variables. Figs. 5(a) and 5(b) show that, similar to the previous benchmark case, the size of quality step is increasing in the inflation rate, whereas the innovation arrival rate is decreasing in it; these results are consistent with the implications of Proposition 1. However, the growth rate of output is now a monotonically decreasing function in inflation rate as described in Fig. 6(a). Recalling the analysis in Section 3.2, when the CIA constraint on manufacturing is present, the growth-promoting effect of higher inflation is two-fold as follows: (a) higher inflation reduces the monopoly profit, which tends to induce entrepreneurs to pursue a more radical innovation; (b) this more radical innovation reallocates labor from the intermediate-good sector to the R&D sector, which tends to raise the innovation arrival rate. When the CIA constraint on manufacturing is absent, these two layers of the positive growth force are significantly weakened, leading the monotonically decreasing effect of inflation on economic growth to take the dominant position. Furthermore, the welfare level continues to be decreasing in inflation, as shown in Fig. 6(b).

Next, we examine the robustness of quantitative results under $f(\lambda) = \lambda^3$, while keeping the benchmark parameter values unchanged. The results regarding the impacts of inflation on the size of quality increment, the arrival rate of innovation, the economic growth rate and the social welfare are reported in Figs. 7(a), 7(b), 8(a) and 8(b), respectively. It is shown that the patterns of our model results are robust to this functional form change. For example, raising the inflation rate still increases the quality step size and decreases both the innovation arrival rate and welfare level. Moreover, despite of a larger threshold value of inflation rate (i.e., 10.3%), the growth rate of output continues to be a hump-shaped function of the inflation rate.

4.3. An extension of a CIA constraint on R&D

When entrepreneurs' R&D activities are constrained by cash, we follow Chu and Cozzi (2014) to assume that entrepreneurs borrow from households to facilitate the wage payment for R&D labor and make returns based on the nominal interest rate i. In this case, the R&D cost for a typical firm $\omega \in [0, 1]$ is given by $\mu_t(\omega)f(\lambda)w_t(1+\eta i)$, where $0 \le \eta \le 1$ represents the degree of the CIA constraint on R&D. Accordingly, the two first-order conditions in (12) and (13) now are given by

$$v_t'(\lambda) = f'(\lambda)w_t(1 + \eta i), \tag{27}$$

$$v_t(\lambda) = f(\lambda)w_t(1 + \eta i). \tag{28}$$

After some manipulations, we can derive the output-to-wage ratio, the aggregate demand for manufacturing labor, and the aggregate labor supply such that[16]

$$\frac{y_t}{w_t} = \frac{(\rho + \mu)f(\lambda)(1 + \eta i) + \kappa}{(\lambda - 1)/\lambda}, \tag{29}$$

$$L_x = \kappa + \frac{(\rho + \mu)(1 + \eta i)f(\lambda) + \kappa}{(\lambda - 1)}, \tag{30}$$

$$L = 1 - \theta(1 + \xi i)\frac{(\rho + \mu)(1 + \eta i)f(\lambda) + \kappa}{(\lambda - 1)/\lambda}. \tag{31}$$

Solving the model yields the two steady-state conditions for λ and μ such that

$$\mu = \frac{\frac{(1-\kappa)(\lambda-1)}{f(\lambda)} - \left[\frac{\kappa}{f(\lambda)} + \rho(1 + \eta i)\right][1 + \theta\lambda(1 + \xi i)]}{\eta i + \lambda + \theta\lambda(1 + \xi i)(1 + \eta i)}, \tag{32}$$

$$\mu = \frac{\kappa/\epsilon}{(\lambda - 1 - 1/\epsilon)f(\lambda)(1 + \eta i)} - \rho. \tag{33}$$

As shown in Fig. 9, a higher nominal interest rate shifts down both the labor-condition and R&D-condition curves. Therefore, the innovation arrival rate is lowered unambiguously. However, the impact on the size of quality increment can be either positive or negative.

Similar to the previous exercises, we resort to a quantitative analysis to evaluate the effects of inflation on the quality step size, the innovation arrival rate, economic growth, and social welfare, respectively, in this extension. We recalibrate this extended model to pin down the value of the parameter η. In addition to the moments used in the benchmark, we use the R&D labor share in the US for calibration. Specifically, we use the ratio of scientists and engineers engaged in R&D over the manufacturing labor force, which is around 4.2%.[17] The calibrated parameter values are reported in Table 2.

Table 2
Parameter values.

ρ	ξ	κ	θ	β	η
0.02	0.17	0.0226	1.9108	0.1504	0.3664

Given the above recalibrated parameters, we quantify the effects of inflation on the aggregate variables. In the presence of CIA constraints on both consumption expenditure and innovative activities, Fig. 10(a) shows that the size of quality increment is still increasing in inflation. In addition, the innovation arrival rate remains to be a decreasing function of inflation, as described in Fig. 10(b). Intuitively, when the CIA constraint on R&D is present, a higher nominal interest rate (and inflation rate) generates an additional negative impact on the innovation arrival rate, since

16 To pinpoint how incorporating CIA constraint on R&D affects the model results, we do not consider the CIA constraint on manufacturing in this extension for simplicity.

17 The number of scientists and engineers engaged in R&D is obtained from Science and Engineering Indicators 2000 (Appendix Tables 3–25) published by the National Science Foundation. The data on manufacturing employees are obtained from the Bureau of Labor Statistics.

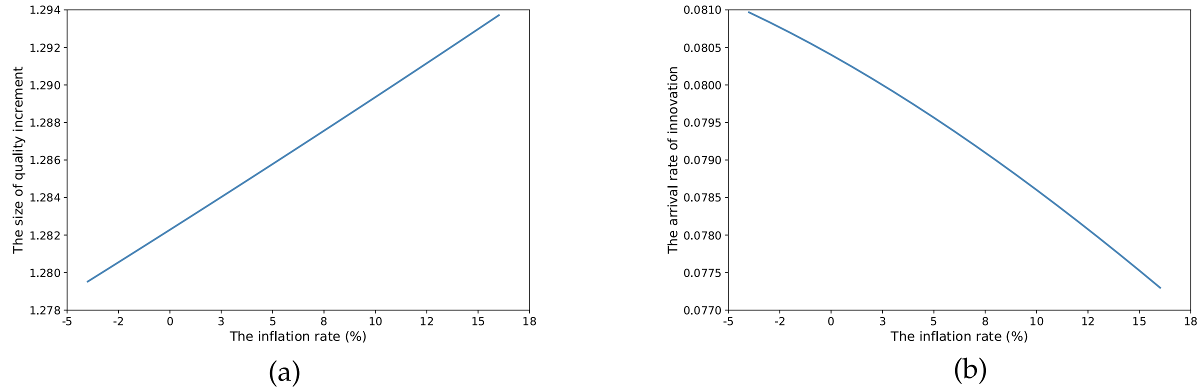

Fig. 3. (a) Inflation and size of quality increment; (b) Inflation and arrival rate of innovation.

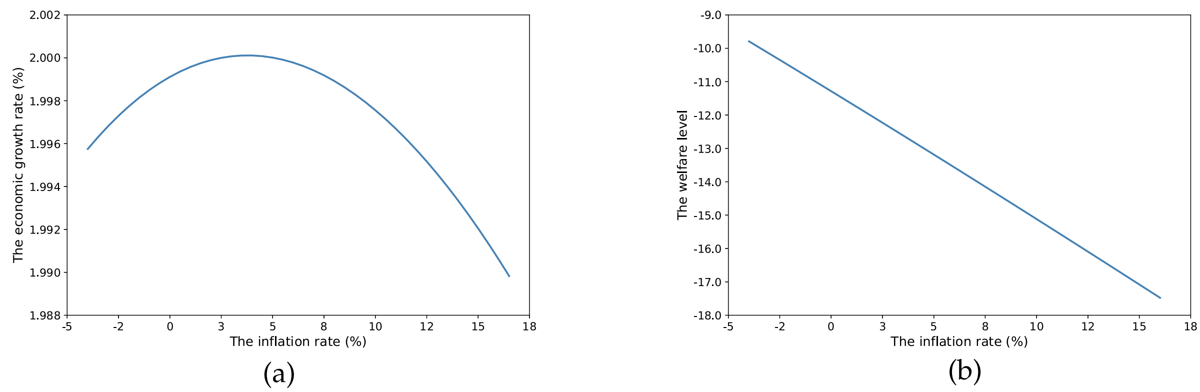

Fig. 4. (a) Inflation and economic growth; (b) Inflation and social welfare.

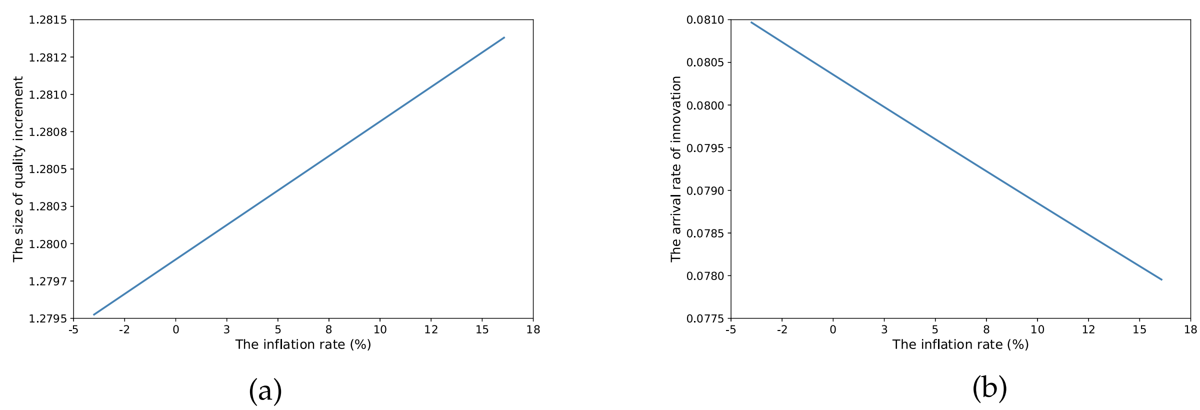

Fig. 5. (a) Inflation and size of quality increment ($\alpha = 0$); (b) Inflation and arrival rate of innovation ($\alpha = 0$).

the increase in the R&D cost discourages R&D incentives. Moreover, the lowered R&D labor demand mitigates the rise in the real wage rate and weakens the impact of the nominal interest rate on the monopoly profit, which in turn lessens the positive growth effect due to a large quality increment. Therefore, a higher inflation rate results in a lower economic growth rate. Fig. 11(a) shows that raising the nominal interest rate from 0 to 20 percentage point causes a decline in the economic growth rate by 10.996% (percentage), and this magnitude is larger than the one in the benchmark case (i.e., 0.259%). As for the welfare effect of inflation, Fig. 11(b) indicates that the Friedman rule still leads to a socially optimal outcome.

The last exercise is to explore the impacts of inflation in a case where consumption, manufacturing and R&D activities are all subject to the CIA constraint. In this general case, the two steady-state conditions for λ and μ solving the model are given by

$$\mu = \frac{\frac{(1-\kappa)(\lambda-1)(1+\alpha i)}{f(\lambda)} - \left[\frac{\kappa(1+\alpha i)}{f(\lambda)} + \rho(1+\eta i)\right][1+\theta\lambda(1+\xi i)(1+\alpha i)]}{(1+\eta)i + (\lambda-1)(1+\alpha i) + \theta\lambda(1+\xi i)(1+\alpha i)(1+\eta i)}, \tag{34}$$

$$\mu = \frac{\kappa(1+\alpha i)/\epsilon}{(\lambda - 1 - 1/\epsilon)f(\lambda)(1+\eta i)} - \rho. \tag{35}$$

Again, we numerically evaluate the effects of a higher inflation rate. Interestingly, we find that when setting $\eta = 0.012$ and preserving other parameter values as listed in Table 1, the influences of inflation on the size of quality increment, the arrival rate of innovation, the economic growth rate, and the welfare level are similar to the counterparts in the benchmark case, as

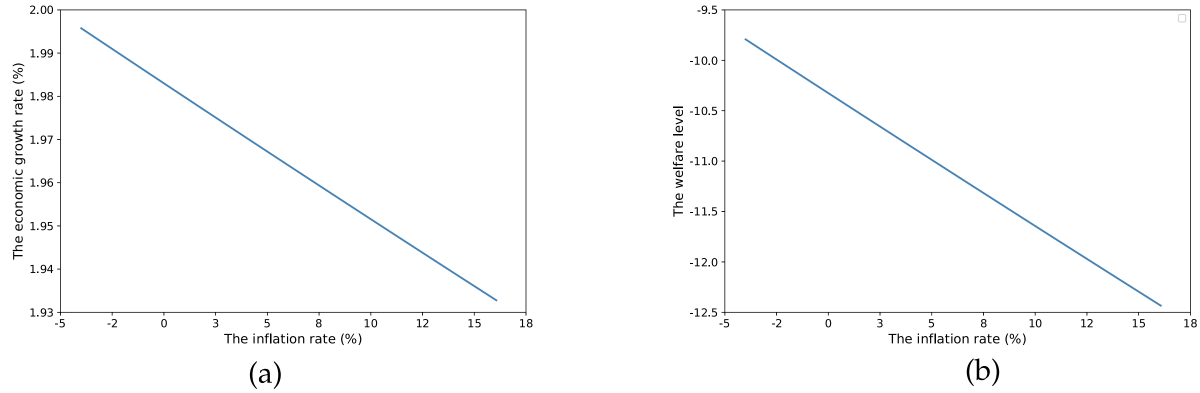

Fig. 6. (a) Inflation and economic growth ($\alpha = 0$); (b) Inflation and social welfare ($\alpha = 0$).

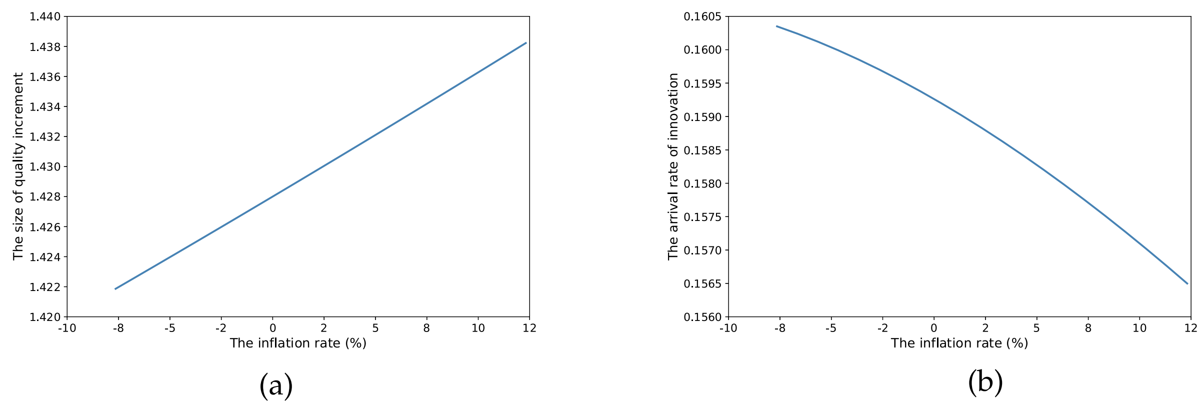

Fig. 7. (a) Inflation and size of quality increment ($f(\lambda) = \lambda^3$); (b) Inflation and arrival rate of innovation ($f(\lambda) = \lambda^3$).

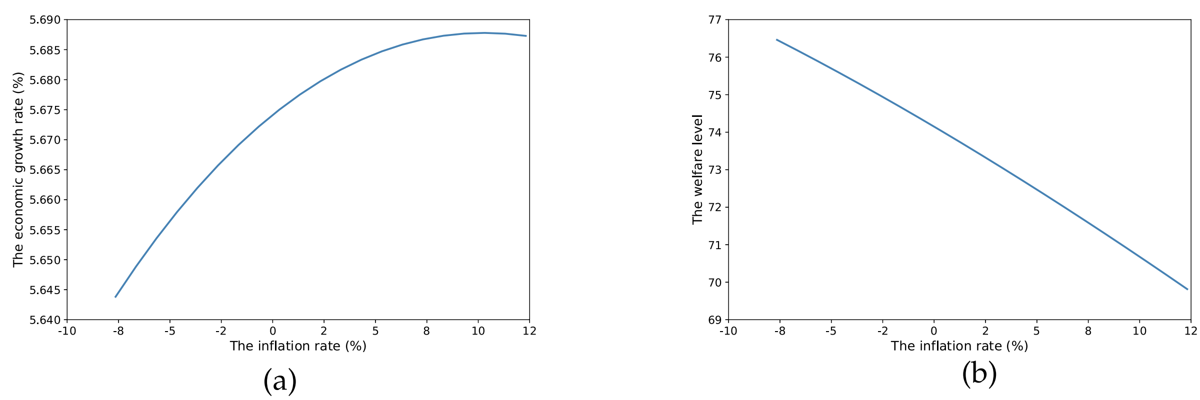

Fig. 8. (a) Inflation and economic growth ($f(\lambda) = \lambda^3$); (b) Inflation and social welfare ($f(\lambda) = \lambda^3$).

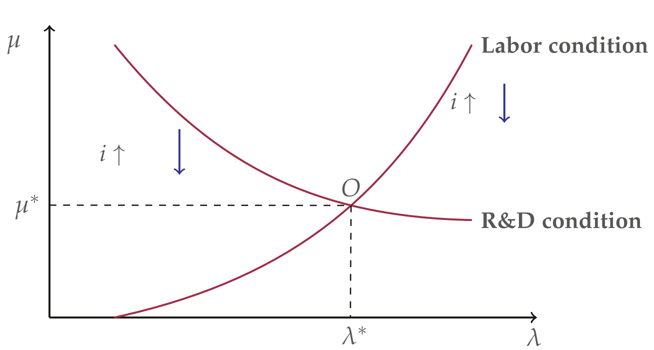

Fig. 9. The steady-state equilibrium under CIA constraints on consumption and R&D.

shown in Figs. 12(a), 12(b), 13(a), and 13(b). In particular, raising the nominal interest rate from 0 to 0.2 increases the quality step size of innovation by 1.11%, and decreases the arrival rate of innovation by 4.941%. The growth effect of inflation in this circumstance is still hump-shaped as in the benchmark case, but with lower growth-maximizing inflation rate at 2.14%. It is consistent with cross-country regression results in López-Villavicencio and Mignon (2011) (i.e., 2.7%), and also our empirical findings (i.e., 2%–3%) based on the US economy as described in the next section.

5. Empirical analysis

The analytical part in this model (including the theoretical and numerical analyses) predicts that inflation and long-run growth

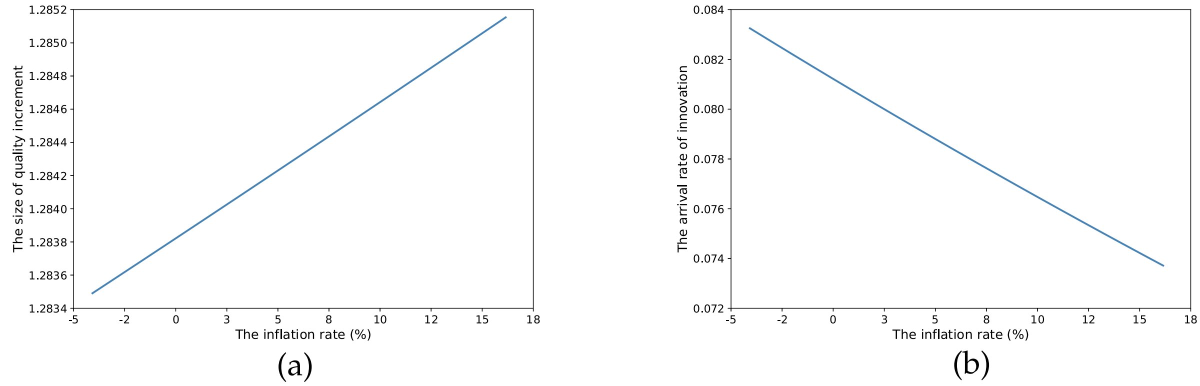

Fig. 10. (a) Inflation and size of quality increment ($\eta = 0.3664$); (b) Inflation and arrival rate of innovation ($\eta = 0.3664$).

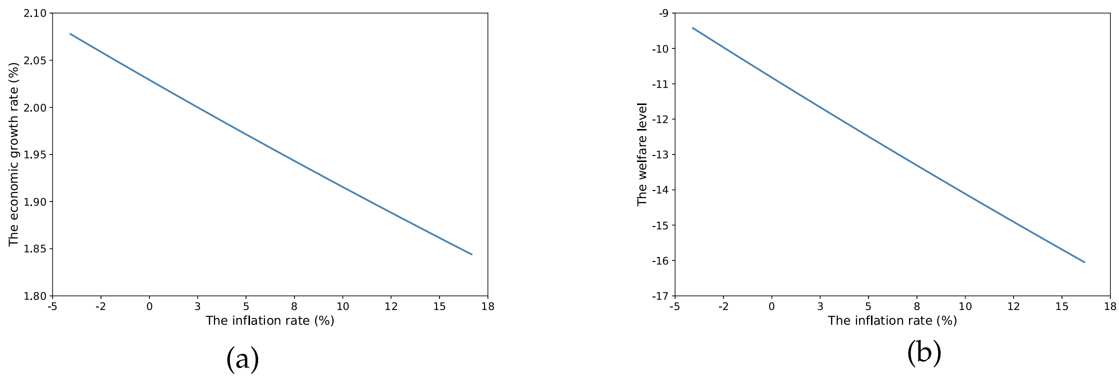

Fig. 11. (a) Inflation and economic growth ($\eta = 0.3664$); (b) Inflation and social welfare ($\eta = 0.3664$).

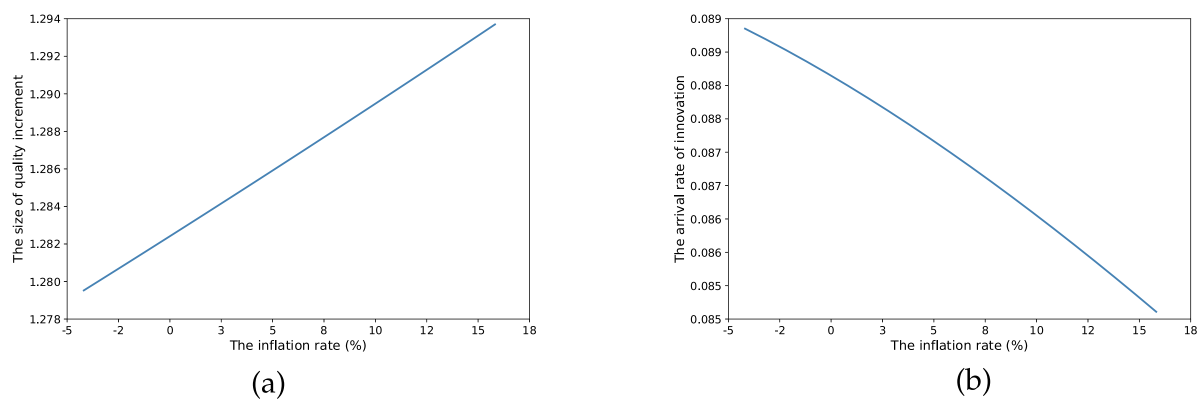

Fig. 12. (a) Inflation and size of quality increment ($\eta = 0.012, \alpha = 1$); (b) Inflation and arrival rate of innovation ($\eta = 0.012, \alpha = 1$).

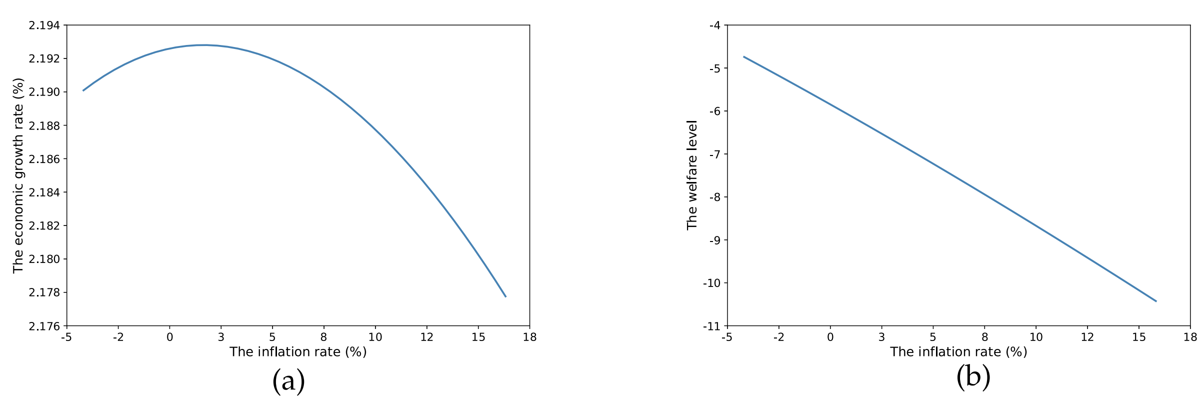

Fig. 13. (a) Inflation and economic growth ($\eta = 0.012, \alpha = 1$); (b) Inflation and social welfare ($\eta = 0.012, \alpha = 1$).

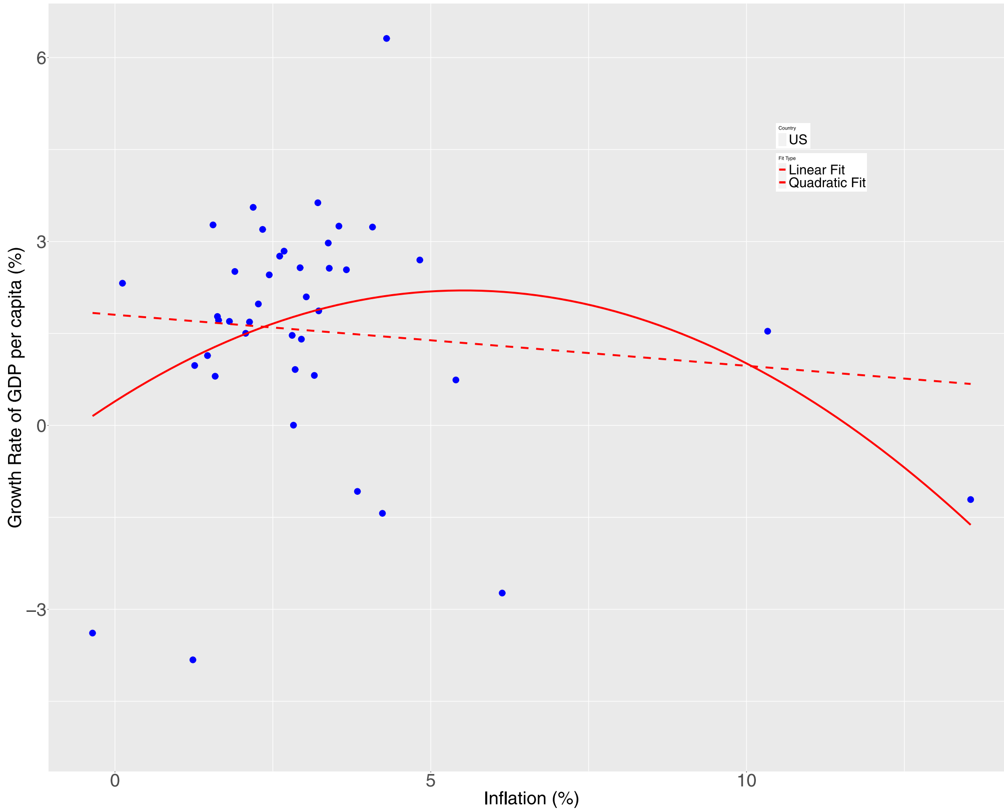

Fig. 14. The relation between inflation and economic growth. Note: Full sample.

may exhibit either a decreasing or an inverted-U relation, depending on the presence of CIA constraints and the functional form of $f(\lambda)$. In this section, we perform an empirical analysis to show which profile between inflation and growth is the most appropriate for the US economy.

Recent studies, such as Khan and Senhadji (2001), Burdekin et al. (2004), López-Villavicencio and Mignon (2011) and Eggoh and Khan (2014), have documented clear empirical evidence on the nonlinear relation between inflation and economic growth. While these studies primarily focus on estimating the threshold level of inflation beyond which inflation generates a substantially different impact on growth, López-Villavicencio and Mignon (2011) explicitly incorporate squared inflation into their regressions, and report that the coefficient estimates (using samples covering various country groups) are negative and statistically significant. Their finding highlights the possibility of an inverted-U shaped relation between inflation and economic growth.

Distinct from previous studies exploiting cross-country panel data, the empirical practice of this paper aims to explore the statistical long-run relation between inflation and growth in the US economy, and hence, restricts attention to the US time series data. Along this line of effort, we estimate the following regression through ordinary least squares (OLS):

$$g_t = \beta_1 \pi_t + \beta_2 \pi_t^2 + HX_{t-1} + \varepsilon_t \tag{36}$$

where g denotes the growth rate of GDP per capita; π denotes inflation; and H is the coefficient matrix on a vector of one-period lagged control variables, X, which includes the capital growth rate, trade openness, the ratio of government expenditure to GDP, and economic freedom. In Eq. (36), squared inflation is introduced to capture the potential nonlinear effect of inflation on growth. Under some alternative model specifications, we also consider to incorporate the time trend as a robustness check.

Our empirical analysis collects yearly US data on seven variables, ranging from 1980 to 2020. Detailed data description is provided in Appendix B. Fig. 14 presents a scatter plot of the US inflation against the growth rate of GDP per capita, and shows that the inverted-U shaped quadratic fit seems to better capture the relation. Notice that the US inflation rates in 1980 and 1981 are 13.55% and 10.33%, respectively, which remarkably exceed the mean of the rest observations (2.73%). To alleviate the concern that the observed hump-shaped inflation-growth relation might be biased, Fig. 15 plots the data after removing the two aforementioned potential outliers. However, the nonlinear effect of inflation on economic growth still seems existent.

Table 3 reports the estimated effect of inflation on economic growth under various model specifications. Under Column (1), it is found that inflation does not have any statistically significant impact on economic growth once squared inflation is excluded, which seems to be consistent with the view of long-run money neutrality. In sharp contrast, however, Columns (2) and (3) show that the point estimates of squared inflation are negative and statistically significant even when the control variables are excluded, regardless of the removal of the potential outliers. Once control variables are incorporated, Columns (4) and (5) suggest a strong inverted-U effect of inflation on growth, with and without controlling for time trend. It is worth mentioning that incorporating economic freedom into the control vector substantially reduces the number of observations, since the data on economic freedom measured by the Fraser Index is only available at the quinquennial frequency prior to 2000. Nevertheless, Column (6) indicates that exploiting more observations by dropping economic freedom from the control vector yields an even stronger inverted-U effect of inflation on the growth rate of GDP per capita. Across all model specifications with squared inflation, the growth-maximizing inflation is found to be around 2% to 3%, which is largely consistent with the theoretical prediction of our quantitative analysis. Notice

Table 3
The Effect of Inflation on Economic Growth — US Data..

	Growth Rate of GDP per capita (%)									
	Full Sample: 1980 to 2020						Before 2008		After 2008	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
π	0.28 (0.83)	0.66 (1.22)	2.32*** (3.07)	2.07** (2.81)	1.89** (2.41)	1.46* (1.76)	−2.12 (−2.10)	5.42 (2.31)	0.51** (3.67)	3.04*** (9.32)
π^2	/	−0.06* (−1.69)	−0.38*** (−3.47)	−0.49** (−2.91)	−0.45** (−2.66)	−0.31*** (−2.75)	/	−1.30* (−3.06)	/	−0.79*** (−8.36)
Growth Rate of Capital	0.37* (2.09)	/	/	0.36* (2.10)	0.41** (2.21)	−0.05 (−0.31)	1.13** (3.54)	0.89* (3.89)	0.24 (0.66)	2.51*** (7.77)
Trade Openness	−0.37*** (−3.68)	/	/	−0.23** (−2.75)	−0.34*** (−3.23)	−0.21*** (−3.39)	0.11 (0.41)	0.08 (0.13)	−0.36* (−2.35)	−0.66** (−8.28)
Gov Spending to GDP Ratio	−0.62 (−1.60)	/	/	−0.10 (−0.28)	0.22 (0.47)	−0.30 (−1.06)	4.52* (2.54)	4.39 (1.69)	−0.80 (−1.32)	5.41** (5.64)
Economic Freedom	−7.12*** (−3.75)	/	/	−4.43** (−2.30)	−3.57* (−1.80)	/	14.17** (2.86)	13.12 (1.79)	−10.69*** (−8.06)	3.75 (1.54)
Time Trend	No	No	No	No	Yes	No	No	Yes	No	Yes
Removal of Outliers	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	21	41	39	21	21	37	10	10	11	11
Adjusted R^2	0.36	0.07	0.23	0.36	0.61	0.35	0.22	0.71	0.80	0.97

Notes: (1) Estimation with Economic Freedom as a control variable reduces the number of observations to 21 due to data availability; (2) t−statistics based on robust standard errors are reported in the parentheses;
***Denotes $p \leq 0.01$.
**Denotes $p \leq 0.05$.
*Denotes $p \leq 0.1$.

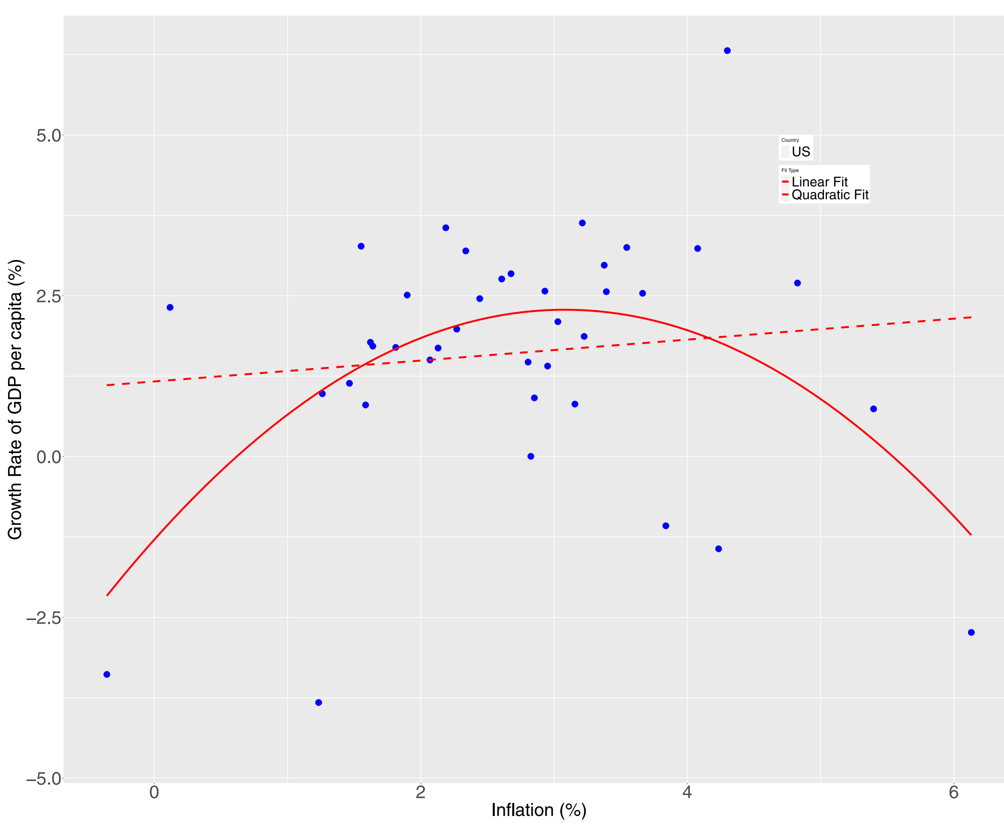

Fig. 15. The relation between inflation and economic growth. Note: Observations with inflation rate above 10% are removed.

that the empirical findings are robust to the estimation using subsamples where periods prior to and after the Great Recession are separately considered.

In summary, our empirical analysis suggests that the relation between inflation and long-run growth in the US is generally inverted-U. Therefore, this stylized fact is well captured by the analytical results of our model, particularly when CIA constraints are imposed on (i) consumption and manufacturing or (ii) all the three channels (i.e., consumption, manufacturing, and R&D).

6. Conclusion

In this study, we analyze the effects of monetary policy on quality increment, innovation, economic growth, and social welfare, respectively. In the model with a CIA constraint only on consumption, we find that a higher nominal interest rate induces R&D firms to pursue a larger quality step size, which tends to stimulate economic growth. Nevertheless, a higher nominal interest rate raises the R&D cost and tends to depress innovation and economic growth. The CIA constraint on manufacturing reinforces the positive growth effect and weakens the negative effect. In contrast, the CIA constraint on R&D enhances only the negative growth effect. By calibrating our model to the US economy, we find that the economic growth rate can be either a monotonically decreasing or hump-shaped function of the inflation rate, whereas the social welfare is always decreasing in inflation. Finally, we show that the hump-shaped relation between inflation and long-run growth is more empirically relevant to the US economy.

This study can be extended in two directions. First, by normalizing the population size to unity, this study sterilizes the strong scale-effect problem present in the first-generation endogenous growth model such as in Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Alternatively, one may remove scale effects in the Schumpeterian growth model by considering the semi-endogenous-growth approach as in Kortum (1997) and Segerstrom (1998) or the second-generation approach as in Peretto (1998) and Howitt (1999). Second, monetary policy in this study is introduced by imposing CIA constraints in different sectors. One may revisit how other formulations that incorporate monetary policy, such as money-in-utility function in Chu and Lai (2013) and price rigidity (via menu costs) in Oikawa and Ueda (2018), will alter the impacts of inflation on nominal macroeconomic variables in a Schumpeterian growth model with endogenous quality increment. We leave these potentially interesting extensions to future research.

Acknowledgements

We are grateful to Juan D. Moreno-Ternero (the Editor), the Associate Editor, and the anonymous Referee for their helpful comments and generous suggestions. Hu gratefully acknowledges financial support from the Start-up Research Grant provided by the Faculty of Social Sciences at University of Macau (Grant No. SRG2018-00134-FSS). Yang gratefully acknowledges financial support from the Asia Pacific Academy of Economics and Management at University of Macau. Zheng gratefully acknowledges financial support from the China Postdoctoral Science Foundation (Grant No. 2021M690434). The usual disclaimer applies.

Appendix A. Proofs

A.1. Proof of Lemma 1

Suppose that a time path of $[i_t]_{t=0}^{\infty}$ is stationary such that $i_t = i$ for all t. Define a transformed variable by $\Phi_t \equiv y_t/v_t$. Therefore, its law of motion is given by

$$\frac{\dot{\Phi}_t}{\Phi_t} = \frac{\dot{y}_t}{y_t} - \frac{\dot{v}_t}{v_t}. \tag{A.1}$$

Using the final-good resource condition $c_t = y_t$ and the Euler equation in (4), the law of motion for y_t is

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{A.2}$$

From (11), the law of motion for v_t is

$$\frac{\dot{v}_t}{v_t} = r_t + \mu_t - \frac{\Pi_t}{v_t}, \tag{A.3}$$

where $\mu_t = L_{r,t}/f(\lambda)$ and Π_t stems from (9). Substituting (A.2) and (A.3) into (A.1) yields

$$\frac{\dot{\Phi}_t}{\Phi_t} = \left(\frac{\lambda - 1}{\lambda}\right)\Phi_t - \frac{\kappa}{f(\lambda)} - \frac{L_{r,t}}{f(\lambda)} - \rho, \tag{A.4}$$

where $v_t = f(\lambda)w_t$ in (13) has been applied. To derive a relationship between $L_{r,t}$ and Φ_t, we first use (13) and (18) to derive

$$L_{x,t} = \kappa + \frac{(y_t/v_t)(v_t/w_t)}{\lambda} = \kappa + \frac{\Phi_t f(\lambda)}{\lambda}. \tag{A.5}$$

In addition, substituting $c_t = y_t$ and (13) into (5) yields

$$L_t = 1 - \theta(1+\xi i)\frac{c_t}{w_t} = 1 - \theta(1+\xi i)\Phi_t f(\lambda). \tag{A.6}$$

Then, substituting (A.5) and (A.6) into the labor-market-clearing condition yields

$$L_{r,t} = L_t - L_{x,t} = 1 - \kappa - f(\lambda)\Phi_t\left[\theta(1+\xi i) + \frac{1}{\lambda}\right]. \tag{A.7}$$

Substituting (A.7) into (A.4) yields an autonomous dynamical equation of Φ_t such that

$$\frac{\dot{\Phi}_t}{\Phi_t} = [1 + \theta(1+\xi i)]\Phi_t - \left[\frac{1}{f(\lambda)} + \rho\right]. \tag{A.8}$$

Given that λ is stationary over time and Φ_t is a control variable, the coefficient associated with Φ_t being positive implies that the dynamics of Φ_t is characterized by saddle-point stability such that Φ_t jumps immediately to its steady-state value given by

$$\Phi = \frac{1/f(\lambda) + \rho}{1 + \theta(1+\xi i)}. \tag{A.9}$$

Eqs. (A.5), (A.6), and (A.7) imply that if Φ is stationary, then L_x, L_r, and L must all be stationary as well.

A.2. Uniqueness of the steady-state equilibrium in Section 3.1

For any given i, differentiating (24) with respect to λ yields

$$\begin{aligned}
&\frac{\partial \mu}{\partial \lambda} \gtrless 0 \\
\Leftrightarrow &\frac{(1-\kappa)\left\{\lambda f(\lambda) - (\lambda - 1)[\lambda f'(\lambda) + f(\lambda)]\right\}}{[\lambda f(\lambda)]^2} \\
&+ \frac{\rho}{\lambda^2} - \frac{\kappa\theta\lambda f(\lambda)(1+\xi i) - \kappa[1 + \theta\lambda(1+\xi i)][\lambda f'(\lambda) + f(\lambda)]}{[\lambda f(\lambda)]^2} \gtrless 0 \\
\Leftrightarrow &\frac{(1-\kappa)[\lambda - (\lambda-1)(1+\epsilon)]}{f(\lambda)} \\
&+ \rho - \frac{\kappa\theta\lambda(1+\xi i) - \kappa[1+\theta\lambda(1+\xi i)](1+\epsilon)}{f(\lambda)} \gtrless 0 \\
\Leftrightarrow &(1-\kappa)[\lambda - (\lambda-1)(1+\epsilon)] + \rho f(\lambda) + \kappa[1 + \epsilon + \theta\lambda\epsilon(1+\xi i)] \gtrless 0 \\
\Leftrightarrow &(1-\kappa)(1 + \epsilon - \lambda\epsilon) + \rho f(\lambda) + \kappa[1 + \epsilon + \theta\lambda\epsilon(1+\xi i)] \gtrless 0 \\
\Leftrightarrow &1 + \epsilon + \rho f(\lambda) + \lambda\epsilon[\kappa - 1 + \kappa\theta(1+\xi i)] \gtrless 0.
\end{aligned} \tag{A.10}$$

Apparently, the left-hand side of the last inequality is an increasing function of κ. As $\kappa \to 1$, the last inequality is reduced to $1 + \epsilon + \rho f(\lambda) + \lambda\epsilon\theta(1+\xi i) > 0$. As $\kappa \to 0$, the last inequality is reduced to $1 + \epsilon + \rho f(\lambda) - \lambda\epsilon > 0$ if $\lambda < 2$, which holds since the value of λ in empirical studies is generally smaller than 2. Therefore, we obtain $\partial\mu/\partial\lambda > 0$. This implies that μ is a monotonically increasing function of λ and features a positive slope and a positive λ-intercept in the $\{\mu, \lambda\}$ space as shown in Fig. 1. Moreover, it is straightforward to verify that (21) implies that μ is a monotonically decreasing function of λ and features a negative slope,[18] with no intercepts in the $\{\mu, \lambda\}$ space as shown in Fig. 1. Therefore, there must exist only one equilibrium in which λ and μ are uniquely determined.

Appendix B. Data description

Yearly US data on the investigated variables are described as follows:

(1) GDP per Capita: GDP per capita annual growth rate (based on constant 2010 US dollars), downloaded from the World Bank Database; Series NY.GDP.PCAP.KD.ZG.

(2) Import Share in GDP: Import values as a percentage of GDP, downloaded from the World Bank Database; Series NE.IMP.GNFS.ZS.

(3) Export Share in GDP: Export values as a percentage of GDP, downloaded from the World Bank Database; Series NE.EXP.GNFS.ZS.

(4) Inflation: Annual percentage change in Consumer Prices, downloaded from the World Bank Database; Series FP.CPI.TOTL.ZG.

(5) Economic Freedom: Fraser Index, extracted from the 2019 Annual Report published by Fraser Institute (https://www.fraserinstitute.org/studies/economic-freedom).

(6) Government Spending to GDP Ratio: General government final consumption expenditure as a percentage of GDP, downloaded from the World Bank Database; Series NE.CON.GOVT.ZS.

(7) Capital Stock: Capital stock at current Purchasing Power Parities (2011 US dollars), downloaded from Penn World Table 9.1

Given the above series, the growth rate of capital is computed as the annual percentage change in capital stock; the degree of trade openness is defined as the sum of import and export shares in GDP; and the Fraser index is used as a measure of economic freedom.

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[18] Eq. (21) shows that as λ approaches $1 + 1/\epsilon$, μ goes to infinity.

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