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Fast communication

On imperfect pricing in globally constrained noncooperative games for cognitive radio networks

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ABSTRACT

Pricing is often used in noncooperative games or Nash equilibrium problems (NEPs) to meet global constraints in cognitive radio networks. In this paper, we analyze the pricing mechanism for a class of solvable NEPs with global constraints, called monotone NEPs. In contrast to the ideal assumption of perfect measure of pricing functions, in practice pricing functions are often imperfectly known and subject to uncertainty. We theoretically analyze the impacts of bounded uncertainty and price-updating step sizes of imperfect pricing in globally constrained NEPs for cognitive radio networks.

1. Introduction

Noncooperative games are also called Nash equilibrium problems (NEPs) [1] that characterize conflicts among interacting decision-makers called players, where each player is regarded to be rational and wishes to selfishly optimize his own payoff. Such game theoretical models have been widely applied to communications and signal processing systems where conflicts or competition are inevitable, for example interference among wireless links (see a special issue [2] on game theory).

The solution to an NEP, i.e., Nash equilibrium (NE), is a point at which no player can gain or achieve a better payoff by unilaterally changing his strategy. In practice, such a solution may be obtained via the best-response algorithms [1], in which players simply optimize their own payoff given the strategies of the others according to a prescribed schedule, e.g. a sequential order. One example is the iterative waterfilling algorithm that arose in power control for digital subscriber lines [3]. However, due to players' selfish behaviors, the NE is often socially inefficient in the sense that global requirements are often unsatisfied.

A common way to improve the social efficiency of the NE is to use pricing that penalizes players' selfish behaviors through some pricing function [4]. As an important application, several pricing mechanisms [5,6] have been proposed for cognitive radio networks (CRNs) where secondary users (SUs) compete the resources of primary users (PUs) but have to satisfy some global interference constraints imposed by PUs. It was shown in [5,6] that the pricing mechanisms can be distributedly implemented and enforce the players (SUs) to meet the global constraints.

The NEP based methods rely on local information measurements, which are, however, often imperfect in CRNs. For example, the channel state information (CSI) between SUs and PUs [7], the interference plus noise (IPN)

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[8], or the best response [9] could be imperfectly measured by SUs. In particular, [10,11] considered imperfect SU-to-PU CSI in pricing NEP designs for CRNs. Although these works considered imperfect information measurements by SUs, they all assumed that pricing functions can be perfectly measured by PUs. In practice, however, pricing functions are more likely to be imperfectly measured since any imperfect local measurement (of, e.g., CSI) by SUs could lead to imperfect measurement of pricing functions by PUs. To the best of our knowledge, imperfect pricing in globally constrained NEPs for CRNs has not been addressed yet.[1]

In this paper, we would like to investigate the influence of imperfect pricing in a class of solvable NEPs called monotone NEPs [13] with achievable solutions by the best-response algorithms. The monotonicity leads to a favorable property called co-coercivity that facilitates the NEPs to meet global constraints with perfect pricing. Then, we consider a more practical situation where pricing functions are imperfectly measured and subject to bounded uncertainty. We theoretically analyze the global impacts of bounded uncertainty and choices of step sizes for the pricing updating mechanism in globally constrained NEPs. The studied framework is then demonstrated through numerical examples in CRNs.

2. Nash equilibrium problem with pricing for CRNs

Consider a CRN of K PUs and N SUs over L-subcarrier interference channels. Let h_{ji}^l be the channel between the secondary transmitter j and the secondary receiver i on subcarrier l, and g_{ik}^l be the channel between the transmitter of SU i and the receiver of PU k on subcarrier l. Let $\mathbf{p}_i = (p_i^l)_{l=1}^L$ with p_i^l being the power allocated by SU i on subcarrier l. Then, the information rate of SU i is given by

$$r_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \sum_{l=1}^{L} \log\left(1 + \frac{|h_{ii}^l|^2 p_i^l}{\sigma_i^l + \sum_{j\neq i} |h_{ji}^l|^2 p_j^l}\right), \tag{1}$$

where σ_i^l is the noise power on subcarrier l. Observe that $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$ depends not only on SU i's transmit power $\mathbf{p}_i$ but also on the transmit power $\mathbf{p}_{-i} = (\mathbf{p}_j)_{j\neq i}$ of the other SUs. A popular way to design strategies of cocurrent transmission of all SUs is to exploit noncooperative game, also known as NEP.

An NEP consists of three components [1]: players, payoff (or cost) functions, and strategy sets. Here, the players are $i = 1, \ldots, N$ SUs and the payoff function of player (or SU) i is his information rate $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$. The strategy set of player i is given by $\mathcal{P}_i = \{\mathbf{p}_i : \sum_{l=1}^L p_i^l \le P_i\}$, which limits the transmit power of SU i below P_i. Then, in the NEP, each player i would aim to maximize his information rate $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$ by choosing a proper power strategy from $\mathcal{P}_i$. The solution to the NEP, also known as Nash Equilibrium (NE), is a strategy profile $\mathbf{p} = (\mathbf{p}_i)_{i=1}^N$, at which no player can gain or achieve a larger rate by unilaterally changing his strategy.

[1] In the case that payoff functions are not fully known, learning mechanisms can be used [12].

The above-mentioned (non-priced) NEP is built on the selfish nature of the players and thus may lead to socially inefficient NE in the sense that, at the NE, either some global constraint is violated or overall system performance is not good. Specifically, to protect PUs' communications in the CRN, the SUs must satisfy the global interference constraints

$$\sum_{i=1}^{N} \sum_{l=1}^{L} |g_{ik}^l|^2 p_i^l \le I_k, \quad k = 1, \ldots, K \tag{2}$$

which restrict the interference caused by all SUs at each PU k below the given threshold I_k. Each SU selfishly optimizing his own payoff would lead to violations of the global interference constraints.

An effective way to tackle this issue is to introduce pricing into NEPs and properly penalize players' selfish behaviors. For each PU k, we can define the pricing function $z_k(\mathbf{p}) = \sum_{i=1}^N \sum_{l=1}^L |g_{ik}^l|^2 p_i^l - I_k$, and associate each pricing function $z_k(\mathbf{p})$ with a price $\lambda_k \ge 0$. Then, the priced NEP can be mathematically formulated as

$$(\mathcal{G}_\lambda): \underset{\mathbf{p}_i \in \mathcal{P}_i}{\text{maximize}}\ r_i(\mathbf{p}_i, \mathbf{p}_{-i}) - \boldsymbol{\lambda}^T \mathbf{z}(\mathbf{p}), \quad \forall i \tag{3}$$

where $\mathbf{z}(\mathbf{p}) = (z_k(\mathbf{p}))_{k=1}^K$ and $\boldsymbol{\lambda} = (\lambda_k)_{k=1}^K$. One can naturally interpret λ_k as the price of violating the interference constraint $z_k(\mathbf{p}) \le 0$. Let $g_i(\mathbf{p}_i, \mathbf{p}_{-i}) = r_i(\mathbf{p}_i, \mathbf{p}_{-i}) - \boldsymbol{\lambda}^T \mathbf{z}(\mathbf{p})$. Then, the NE of $\mathcal{G}_\lambda$ is given by a point $\mathbf{p}^*$ such that $g_i(\mathbf{p}_i^*, \mathbf{p}_{-i}^*) \ge g_i(\mathbf{p}_i, \mathbf{p}_{-i}^*)$, $\forall \mathbf{p}_i \in \mathcal{P}_i$ for $i = 1, \ldots, N$. By properly choosing $\boldsymbol{\lambda}$, the global interference constraints can be satisfied at the NE $\mathbf{p}^*$. Therefore, one shall expect

$$\boldsymbol{\lambda} \ge \mathbf{0}, \quad \mathbf{z}(\mathbf{p}^*) \le \mathbf{0}, \quad \boldsymbol{\lambda}^T \mathbf{z}(\mathbf{p}^*) = 0 \tag{4}$$

where the last condition simply says if the interference constraint is satisfied then no pricing is needed. We term $\mathcal{G}_\lambda$ and (4) a priced NEP and $(\boldsymbol{\lambda}^*, \mathbf{p}^*)$ the pricing equilibrium (PE) if the price vector $\boldsymbol{\lambda}^*$ satisfying (4) at the NE $\mathbf{p}^*$ of $\mathcal{G}_{\lambda^*}$. It was shown in [5,6] that the priced NEP approach leads to a nice distributed network design.

3. Best-response and pricing algorithms

Searching the PE of a priced NEP includes actually two parts: choose proper prices $\boldsymbol{\lambda}$ and find the NE of $\mathcal{G}_\lambda$ with given $\boldsymbol{\lambda}$, both depending on the properties of the strategy sets, the payoff functions, and the pricing functions. For the considered CRN we have the following properties: (1) $\mathcal{P}_i$ is a convex compact set; (2) $r_i(\mathbf{p}_i, \mathbf{p}_{-i})$ is twice differentiable and convex in $\mathbf{p}_i$ for $\forall i$; (3) $z_k(\mathbf{p})$ is convex in $\mathbf{p}$ for $\forall k$. We also introduce $\mathbf{F}(\mathbf{p}) = \left(-\nabla_{\mathbf{p}_i} r(\mathbf{p})\right)_{i=1}^N$ and $\mathcal{P} = \prod_{i=1}^N \mathcal{P}_i$, where $\nabla_{\mathbf{p}_i} r(\mathbf{p})$ is the gradient of $r(\mathbf{p})$ with respect to $\mathbf{p}_i$. With the above properties, given any $\boldsymbol{\lambda} \ge \mathbf{0}$ the NEP $\mathcal{G}_\lambda$ is guaranteed to possess at least one solution [5].

To solve the NEP $\mathcal{G}_\lambda$ with given $\boldsymbol{\lambda}$, we introduce an important concept called the strong monotonicity: $F(\mathbf{p})$ is strongly monotone on $\mathcal{P}$ if $(\mathbf{x}-\mathbf{y})^T (F(\mathbf{x}) - F(\mathbf{y})) \ge \alpha_s \|\mathbf{x}-\mathbf{y}\|^2$ for $\forall \mathbf{x}, \mathbf{y} \in \mathcal{P}$ with a positive constant α_s. To verify the strong monotonicity of $F(\mathbf{p})$, let us define an $N \times N$ matrix $\boldsymbol{\Phi}$ with $[\boldsymbol{\Phi}]_{ij} = -\sup_{\mathbf{p}\in\mathcal{P}} \| -\nabla^2_{\mathbf{p}_i \mathbf{p}_j} r(\mathbf{p}) \|_2$ for $i \ne j$ and $[\boldsymbol{\Phi}]_{ii} = \inf_{\mathbf{p}\in\mathcal{P}} \lambda_{\min}(-\nabla^2_{\mathbf{p}_i} r(\mathbf{p}))$, where $\|\cdot\|_2$ de notes the spectral norm and $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a Hermitian matrix. Then, the strong monotonicity of $F(\mathbf{p})$ is implied by [13]: (C1) $\boldsymbol{\Phi} \succ \mathbf{0}$. Note that (C1) also guarantees the solvability of the NEP $\mathcal{G}_\lambda$ as follows (the proof is omitted and the interested reader is referred to [13] for more details).

Lemma 1. *Suppose that (C1) holds. Then, $F(\mathbf{p})$ is strongly monotone on $\mathcal{P}$, and given any $\boldsymbol{\lambda} \ge \mathbf{0}$ the NEP $\mathcal{G}_\lambda$ has a unique NE, which can be obtained by the best-response algorithm.*

Remark. The best-response algorithm (BRA) simply lets each player minimize his (priced) cost given the strategies of the others in a prescribed order. For example, at each iteration t the players could simultaneously update their strategies $\mathbf{p}_i^{(t+1)} = \arg\max_{\mathbf{p}_i \in \mathcal{P}_i} g_i(\mathbf{p}_i, \mathbf{p}_{-i}^{(t)})$, which, in the CRN, is a convex problem and can be efficiently solved. From Lemma 1, the BRA is guaranteed to converge to the NE of $\mathcal{G}_\lambda$ provided $\boldsymbol{\Phi} \succ \mathbf{0}$. In CRNs, the BRA can be implemented in a fully distributed manner since each SU can locally measure the interference caused by the other SUs. It has been verified [7] that $\boldsymbol{\Phi} \succ \mathbf{0}$ is implied by $\boldsymbol{\Psi} \succ \mathbf{0}$ with $[\boldsymbol{\Psi}]_{ii} = \min_l |h_{ii}^l|^4 / (\sigma_i^l + \sum_{m=1}^N |h_{mi}^l|^2 P_m)^2$ and $[\boldsymbol{\Psi}]_{ij} = -\max_l |h_{ii}^l|^2 |h_{ji}^l|^2 / (\sigma_i^l)^2$ for $i \ne j$, which physically says that the cross interference among the SUs is not very strong [5–7]. Upon the solvability of $\mathcal{G}_\lambda$, the left question is how to set $\boldsymbol{\lambda}$ such the conditions in (4) are satisfied. Let $\mathbf{p}^*(\boldsymbol{\lambda})$ denote the NE of $\mathcal{G}_\lambda$ with given $\boldsymbol{\lambda}$ and let $\mathbf{z}(\boldsymbol{\lambda}) = \mathbf{z}(\mathbf{p}^*(\boldsymbol{\lambda}))$. Then, a simple but effective algorithm to optimize $\boldsymbol{\lambda}$ is as follows:

$$\text{(T1)}: \boldsymbol{\lambda}^{(n+1)} = \left[\boldsymbol{\lambda}^{(n)} + s_n \mathbf{z}\left(\boldsymbol{\lambda}^{(n)}\right)\right]_+ \tag{5}$$

where $[\cdot]_+$ denote the projection onto the nonnegative orthant and s_n is a positive step size. This algorithm admits a natural interpretation: if at iteration n the global constraint is violated, i.e., $\mathbf{z}(\boldsymbol{\lambda}^{(n)}) > 0$, then the players shall be penalized with higher prices at the next iteration. Its convergence depends on the following property of $\mathbf{z}(\boldsymbol{\lambda})$ (the proof is omitted and the interested reader is referred to [14] for more details).

Lemma 2. *Suppose that (C1) holds. Then, $-\mathbf{z}(\boldsymbol{\lambda})$ is co-coercive in $\boldsymbol{\lambda}$, i.e., $(\boldsymbol{\lambda} - \boldsymbol{\eta})^T (\mathbf{z}(\boldsymbol{\eta}) - \mathbf{z}(\boldsymbol{\lambda})) \ge \alpha_c \|\mathbf{z}(\boldsymbol{\lambda}) - \mathbf{z}(\boldsymbol{\eta})\|^2$ for $\forall \boldsymbol{\lambda}, \boldsymbol{\eta} \ge \mathbf{0}$ with a positive constant α_c. Given $0 < s_n < 2\alpha_c$, T1 converges to a point satisfying the conditions in (4).*

Remark. Lemmas 1 and 2 indicate that the PE of a priced NEP can be achieved by T1 with the BRA embedded. Particularly, in CRNs, T1 can be distributedly implemented. Indeed, since each PU k may locally measure the interference generated by all SUs or equivalently $z_k(\boldsymbol{\lambda}^{(n)})$, he can individually update $\lambda_k^{(n+1)}$ according to T1 and broadcast it to all SUs, while the SUs use the BRA to solve the NEP $\mathcal{G}_\lambda$ with given prices also in a distributed way.

4. Imperfect pricing with bounded uncertainty

A pivot in searching the PE is that the pricing function $\mathbf{z}(\boldsymbol{\lambda})$ is known. However, in many practical situations, $\mathbf{z}(\boldsymbol{\lambda})$ can only be imperfectly measured. For example, in CRNs, PUs may not accurately measure the interference caused by SUs exactly after the BRA converges. It is also difficult for SUs to perfectly know the channels g_{ik}^l to PUs, so the interference that SUs mean to generate may not match the interference measured by PUs [7]. Therefore, it is worthwhile to investigate the impact of imperfect pricing in pricing NEPs.

For this purpose, we consider a disturbed pricing function $\hat{\mathbf{z}}(\boldsymbol{\lambda}^{(n)}) = \mathbf{z}(\boldsymbol{\lambda}^{(n)}) + \mathbf{u}_n$ at each iteration of the price updating, where $\mathbf{u}_n$ is a disturbance or error vector containing bounded uncertainty. Therefore, the practical price updating follows:

$$\text{(T2)}: \boldsymbol{\lambda}^{(n+1)} = \left[\boldsymbol{\lambda}^{(n)} + s_n \hat{\mathbf{z}}(\boldsymbol{\lambda}^{(n)})\right]_+. \tag{6}$$

Note that the goal of using pricing in NEPs is to meet the global constraint $\mathbf{z}(\boldsymbol{\lambda}) \le \mathbf{0}$. This can be achieved by T1 using perfect measure of $\mathbf{z}(\boldsymbol{\lambda})$, but may not be exactly satisfied by the practical algorithm T2 using imperfect pricing. Therefore, we shall study the influence of imperfect pricing.

4.1. Diminishing uncertainty

We first investigate the impact of diminishing uncertainty, i.e., $\lim_{n\to\infty} \|\mathbf{u}_n\| = 0$. Let $\mathbf{z}(\boldsymbol{\lambda}) = \mathbf{z}(\mathbf{p}^*(\boldsymbol{\lambda}))$ with $\mathbf{p}^*(\boldsymbol{\lambda})$ being the NE of $\mathcal{G}_\lambda$. Let $(\boldsymbol{\lambda}^*, \mathbf{p}^*)$ be the PE of the pricing NEP obtained by T1 and $\{\boldsymbol{\lambda}^{(n)}\}_{n=0}^\infty$ be the sequence generated by T2. Then, we have the following result.

Theorem 1. *Suppose that (C1) holds, $0 < s_n < 2\alpha_c$, and $\lim_{n\to\infty} \|\mathbf{u}_n\| = 0$. Then, $\liminf_{n\to\infty} \|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\| = 0$.*

Proof. It follows from Algorithms T1 and T2 that

$$\begin{aligned} \|\boldsymbol{\lambda}^{(n+1)} - \boldsymbol{\lambda}^*\|^2 &= \|[\boldsymbol{\lambda}^{(n)} + s_n \hat{\mathbf{z}}\left(\boldsymbol{\lambda}^{(n)}\right)]_+ - [\boldsymbol{\lambda}^* + s_n \mathbf{z}(\boldsymbol{\lambda}^*)]_+\|^2 \\ &\le \|\boldsymbol{\lambda}^{(n)} + s_n \hat{\mathbf{z}}(\boldsymbol{\lambda}^{(n)}) - \boldsymbol{\lambda}^* - s_n \mathbf{z}(\boldsymbol{\lambda}^*)\|^2 \\ &= \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^* + s_n(\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)) + s_n \mathbf{u}_n\|^2 \\ &\le \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^* + s_n(\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*))\|^2 + s_n^2 \|\mathbf{u}_n\|^2 \\ &\quad + 2s_n \|\mathbf{u}_n\| \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^* + s_n(\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*))\| \end{aligned} \tag{7}$$

where the first inequality follows from the nonexpansive property of the nonnegative projection. Using the co-coercive property of $\mathbf{z}(\boldsymbol{\lambda})$ in Proposition 2, we have $(\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*)^T (\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)) \le -\alpha_c \|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\|^2$ and thus

$$\begin{aligned} &\|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^* + s_n(\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*))\|^2 \\ &\quad = \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|^2 + s_n^2 \|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\|^2 \\ &\qquad + 2s_n (\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*)^T (\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)) \\ &\quad \le \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|^2 - s_n(2\alpha_c - s_n) \|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\|^2. \end{aligned} \tag{8}$$

Since $\mathbf{z}(\boldsymbol{\lambda})$ is convex, it is also Lipschitz continuous, i.e., $\|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\| \le \alpha_l \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|$ for some constant $\alpha_l > 0$. Hence, we have

$$\begin{aligned} &\|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^* + s_n(\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*))\| \le \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\| \\ &\quad + s_n \|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\| \le (1 + s_n \alpha_l) \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|. \end{aligned} \tag{9}$$

Combining (7)–(9), we obtain

$$\|\boldsymbol{\lambda}^{(n+1)} - \boldsymbol{\lambda}^*\|^2 \le \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|^2 + s_n w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) \tag{10}$$

where

$$\begin{aligned} w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) = s_n \|\mathbf{u}_n\|^2 - (2\alpha_c - s_n) \|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) \\ - \mathbf{z}(\boldsymbol{\lambda}^*)\|^2 + 2(1 + s_n \alpha_l) \|\mathbf{u}_n\| \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|. \end{aligned} \tag{11}$$

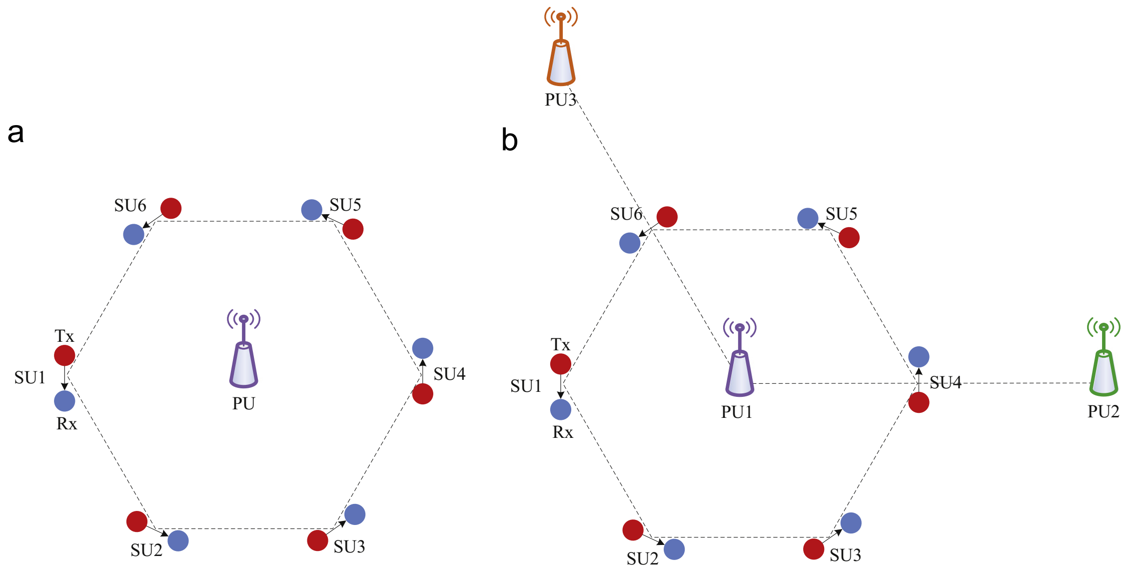

Fig. 1. Two scenarios of CR network: (a) six SUs and one PU and (b) six SUs and three PUs.

Assume $\limsup_{n\to\infty} w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) < 0$. Then, there exists an integer m such that $w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) < 0$ for $\forall n \ge m$. Hence, we have $\|\boldsymbol{\lambda}^{(n+1)} - \boldsymbol{\lambda}^*\|^2 < \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|^2$ for $\forall n \ge m$, which implies that the sequence $\{\|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|^2\}_n$ converges, since it is monotonically decreasing and lower bounded by zero. Then, taking lim sup at both sides of (10), we have $\limsup_{n\to\infty} w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) \ge 0$, which contradicts with the assumption. Thus, we have $\limsup_{n\to\infty} w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) \ge 0$, which implies

$$\liminf_{n\to\infty}(2\alpha_c - s_n)\|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\|^2 \le \limsup_{n\to\infty} s_n\|\mathbf{u}_n\|^2 + 2(1 + s_n\alpha_l)\|\mathbf{u}_n\|\,\|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|. \tag{12}$$

Since $\lim_{n\to\infty}\|\mathbf{u}_n\| = 0$, $0 < s_n < 2\alpha_c$, and $\{\boldsymbol{\lambda}^{(n)}\}_{n=0}^{\infty}$ is bounded, we obtain $\liminf_{n\to\infty}\|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\|^2 = 0$.□

Remark. Theorem 1 implies two consequences. First, the uncertainty, even though diminishing, may have an accumulated influence that impedes T2 from converging exactly. Second, although one cannot expect $\mathbf{z}(\boldsymbol{\lambda}) \le \mathbf{0}$ is exactly satisfied in practice, such a global requirement may be approximately met by T2. Indeed, if at the PE $\mathbf{z}(\boldsymbol{\lambda}^*) = \mathbf{z}(\mathbf{p}^*(\boldsymbol{\lambda}^*)) = \mathbf{0}$ or $\boldsymbol{\lambda}^* > \mathbf{0}$, then Theorem 1 reduces to $\liminf_{n\to\infty}\|\mathbf{z}(\boldsymbol{\lambda}^{(n)})\| = 0$. Thus, given any $n \ge 0$, there always exists $m \ge n$ such that $\mathbf{z}(\boldsymbol{\lambda}^{(m)}) = \mathbf{0}$. This means that the sequence $\{\mathbf{z}(\boldsymbol{\lambda}^{(n)})\}_n$ would fluctuate around zero, and thus the global constraint $\mathbf{z}(\boldsymbol{\lambda}) \le \mathbf{0}$ can be (approximately) satisfied. Indeed, this also implies that when the uncertainty tends to zero, which corresponds to CSI becoming more accurate (by using longer training sequences in channel estimation or using more bits in CSI feedback), the pricing mechanism and NE are still stable.

4.2. Undiminishing uncertainty

In this subsection, we would consider the situation where uncertainty is bounded but not necessarily diminishing. In this case, one may naturally expect that undiminishing uncertainty possibly has a severer impact than diminishing uncertainty. We show that this could be true even if one uses a variable step size, for example a diminishing step size, to be alleviate the influence of undiminishing uncertainty.

Theorem 2. *Suppose that* (A1) *and* (C1) *hold,* $\|\mathbf{u}_n\| \le \varepsilon$, $\lim_{n\to\infty} s_n = 0$, $\sum_{n=0}^{\infty} s_n = \infty$, *and* $\|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\| \le d$. *Then,* $\liminf_{n\to\infty}\|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\| \le \sqrt{\varepsilon d \alpha_c^{-1}}$.

Proof. Recall the definition of $w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n)$ in (10) and assume $\limsup_{n\to\infty} w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) < 0$. Then, there exists an integer m and a positive scalar θ such that $w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) < -\theta$ for $\forall n \ge m$. From (10), we have

$$\begin{aligned}\|\boldsymbol{\lambda}^{(n+1)} - \boldsymbol{\lambda}^*\|^2 &\le \|\boldsymbol{\lambda}^{(n)} - \boldsymbol{\lambda}^*\|^2 - s_n\theta \\ &\le \cdots \le \|\boldsymbol{\lambda}^{(m)} - \boldsymbol{\lambda}^*\|^2 - \sum_{i=m}^{n} s_i\theta.\end{aligned} \tag{13}$$

which, given $\sum_{n=0}^{\infty} s_n = \infty$, implies $\|\boldsymbol{\lambda}^{(n+1)} - \boldsymbol{\lambda}^*\|^2 < 0$ for a sufficiently large n, resulting in a contradiction. Therefore, we have $\limsup_{n\to\infty} w_n(\boldsymbol{\lambda}^{(n)}, \mathbf{u}_n) \ge 0$, which implies (12). Considering that $\|\mathbf{u}_n\| \le \varepsilon$ and $\lim_{n\to\infty} s_n = 0$, one can obtain $\liminf_{n\to\infty}\|\mathbf{z}(\boldsymbol{\lambda}^{(n)}) - \mathbf{z}(\boldsymbol{\lambda}^*)\|^2 \le \varepsilon d \alpha_c^{-1}$.□

Remark. An example of diminishing step sizes is $s_n = s_0(1+c)/(n+c)$, where $s_0 \in (0, 1]$ is the initial step size and $c \ge 0$ is a fixed number. However, Theorem 2 indicates that even if a diminishing step size is adopted, the negative effect of undiminishing uncertainty in the pricing mechanism will not vanish. The (approximate) satisfaction of the global constraint is determined by the error bound. This is in contrast with gradient-projection methods for convex optimization problems, where the impact of bounded uncertainty can often be removed by using diminishing step sizes [15]. The fundamental reason is that the pricing function $\mathbf{z}(\boldsymbol{\lambda})$ is related to $\boldsymbol{\lambda}$ in an indirect and complicated way (through the NE of a game) and the co-coercivity is generally weaker than convexity.

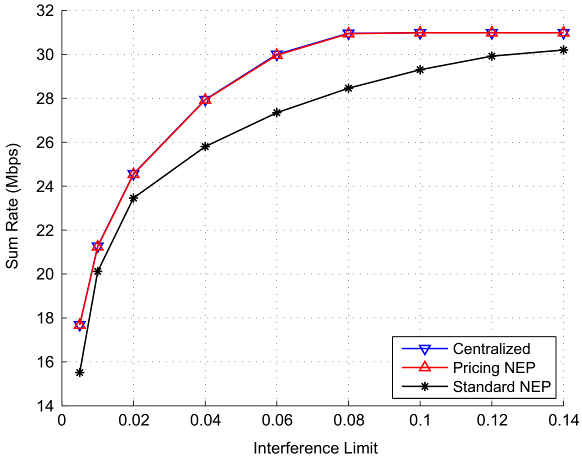

Fig. 2. Sum rates of the centralized, pricing NEP, and standard NEP methods in scenario (a).

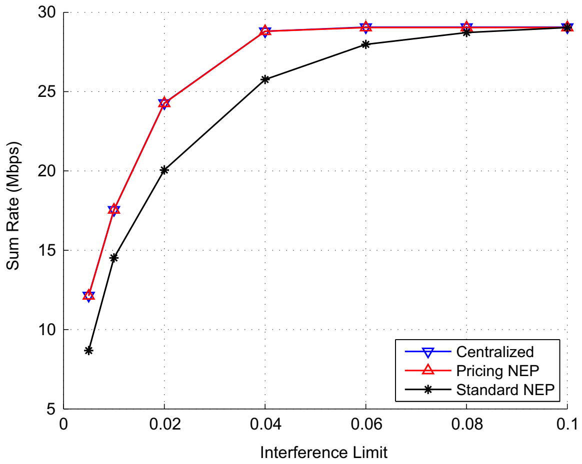

Fig. 3. Sum rates of the centralized, pricing NEP, and standard NEP methods in scenario (b).

5. Numerical examples

We use several numerical examples to demonstrate the pricing NEP framework for the CRN introduced in Section 2. Suppose that there are $N=6$ SUs (players) at the vertices of a hexagonal cell, where each SU is a Tx–Rx link over $L=8$ subcarriers. The distance between each secondary Tx and Rx is identical and used as a unit, and the cell radius is set to be 4 times of a unit. Then, we consider two scenarios as shown in Fig. 1, where in scenario (a) there is $K=1$ base station (PU) at the center of the cell and in scenario (b) there are $K=3$ base stations (PUs). The channels h_{ij}^l and g_{ik}^l are generated according to i.i.d. zero-mean unit-variance Gaussian distributions. All SUs and PUs operate on a bandwidth of 2 MHz. The power budget is set such that $\mathrm{SNR}=P_i/\sigma_i^l$ is at 10 dB.

We first compare the pricing NEP with two different network designs. One is a centralized method that maximizes the sum rate of all SUs subject to the global QoS constraints. The other one is a standard NEP without

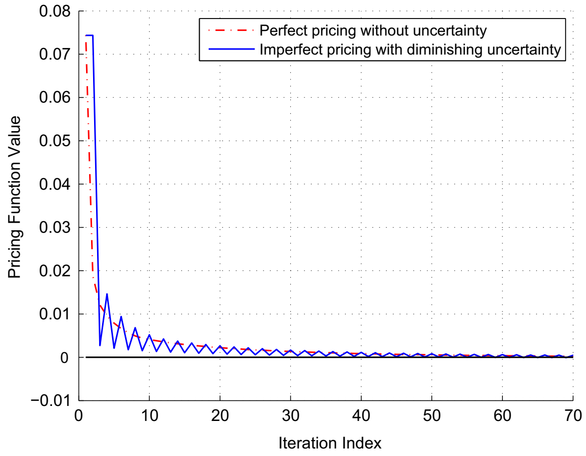

Fig. 4. Iterations of pricing function $z_k(\lambda^{(n)})$ for the perfect pricing (T1) and imperfect pricing (T2) with diminishing uncertainty in scenario (a).

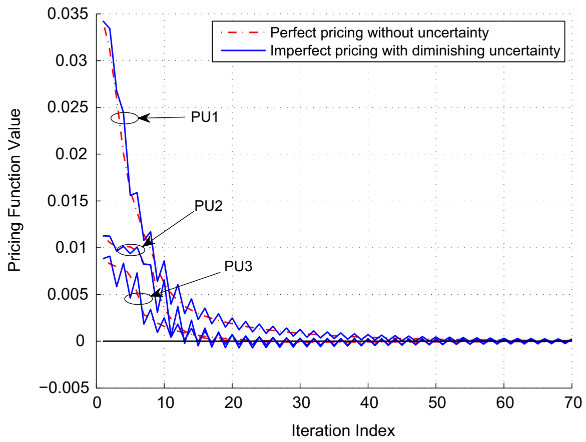

Fig. 5. Iterations of pricing function $z_k(\lambda^{(n)})$ for the perfect pricing (T1) and imperfect pricing (T2) with diminishing uncertainty in scenario (b).

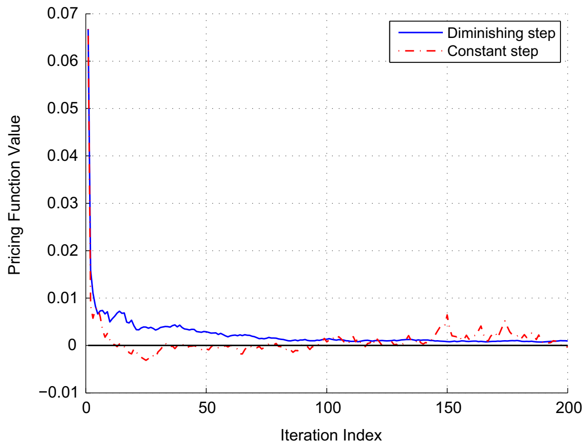

Fig. 6. Iterations of pricing function $z_k(\lambda^{(n)})$ for the imperfect pricing (T2) with undiminishing uncertainty using constant and diminishing step sizes in scenario (a).

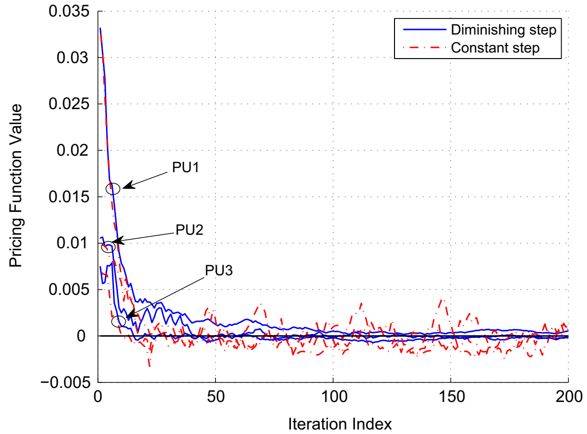

Fig. 7. Iterations of pricing function $z_k(\lambda^{(n)})$ for the imperfect pricing (T2) with undiminishing uncertainty using constant and diminishing step sizes in scenario (b).

pricing, in which each SU is restricted by individual QoS constraints, whose interference limit is set to be I_k/N to guarantee the global QoS for PU k. Figs. 2 and 3 show the sum rates of three methods at different interference limits. Both figures show that the pricing NEP provides almost the same sum-rate performance as the centralized method that generally requires high computational complexity and signalling overhead. The pricing NEP also consistently outperforms the standard NEP, although the latter one does not need price updating. Therefore, the pricing NEP provides a good balance between the global network performance and signaling overhead as well as computational complexity.

Next, in Figs. 4 and 5 we show the iterations of the perfect pricing (T1) and imperfect pricing (T2) with diminishing uncertainty in two scenarios over randomly chosen channel profiles that satisfy (C1). The interference limit is set to be $I=0.01$. The diminishing uncertainty follows $u_n=(-1)^n/10n$ in scenario (a) and $u_n^k=(-1)^n/40n$ for each PU k in scenario (b). One can observe from Figs. 4 and 5 that, for T2, the pricing function $z_k(\lambda^{(n)})= \sum_{i=1}^{N}\sum_{l=1}^{L}|g_{ik}^l|^2 p_i^l(\lambda^{(n)})-I_k$ (for each PU k) fluctuates at the initial stage due to the relatively large uncertainty and finally approaches zero when the uncertainty diminishes. Therefore, the global QoS constraint is satisfied at $z_k(\lambda^{(n)})=0$, which is consistent with Theorem 1.

Finally, in Figs. 6 and 7 we show the iterations of the imperfect pricing (T2) with a diminishing step size as well as a constant step size, where uncertainty u_n is uniformly distributed within $[-0.02, 0.02]$ in scenario (a) and u_n^k is uniformly distributed within $[-0.01, 0.01]$ for each PU k in scenario (b). For both constant and diminishing step sizes, the pricing function $z_k(\lambda^{(n)})$ tends to zero. However, for a constant step size $z_k(\lambda^{(n)})$ keeps vacillating around zero. For a diminishing step size $z_k(\lambda^{(n)})$ steadily approaches zero and then stays within a small range around zero. This is consistent with Theorem 2 that undiminishing uncertainty has an accumulated effect on the pricing mechanism that may not be fully eliminated by diminishing step sizes. However, using diminishing step sizes is still helpful to satisfy the global QoS constraint approximately.

6. Conclusion

We have studied pricing mechanisms with perfect and imperfect measurements of pricing functions in noncooperative games or NEPs to meet global constraints in CRNs. The impacts of bounded uncertainty and price-updating step sizes were investigated. We showed that diminishing and undiminishing uncertainties have a bit different impacts and the effect of the latter one cannot be eliminated even using a diminishing step size. The whole framework was demonstrated through CRN examples.

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